A QUANTUM MECHANICS PRIMER

An Elementary Introduction to
the Formal Theory of
Non-relativistic Quantum Mechanics

by
DANIEL T. GILLESPIE

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Written especially for science majors at the junior level, *A Quantum Mechanics Primer* presents a concise, self-contained, systematic development of the formal theory of quantum mechanics. By carefully simplifying the theory and largely ignoring its many intricate applications, the author manages to convey a remarkably clear and meaningful *perspective* of the quantum theory—a perspective that is often lost in the length and encyclopedic thoroughness of the conventional introductory texts. Thus, the *Primer* is ideal for use either before or in addition to any of the standard elementary quantum mechanics textbooks; its use in this *complementary* sense should help minimize the bewilderment that traditionally accompanies the student's first encounter with quantum mechanics.

A Quantum Mechanics Primer

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To my wife

Louise

Foreword

In this book I have tried to write for science majors at the junior level a self-contained, somewhat simplified but essentially "honest" exposition of the formal theory of quantum mechanics. I have not attempted to treat any of the traditional *applications* of quantum mechanics; instead, I have endeavored only to present a concise, axiomatic development of the *theory*, with a view to bringing out as clearly as possible the main features of its mathematical and conceptual structure.

The book grew out of a series of lectures which I gave to a class of honors sophomore physics majors at Johns Hopkins University in the Spring of 1967. That experiment in an earlier-than-usual introduction to quantum mechanics convinced me that, if one is sufficiently selective and careful, students at roughly this level can be rather quickly brought to a surprisingly good understanding of the formal structure of the quantum theory. This book represents an expansion and rearrangement of those lectures, but the aim and approach are essentially unchanged.

It must be emphasized that this book cannot and is not intended to "supplant" any of the existing texts on elementary quantum mechanics. Its purpose, rather, is to *complement* and *supplement* these texts by providing the student with a simplified but meaningful *perspective* of the theory. To successfully convey such a perspective, one must place a high premium on conciseness as well as clarity; accordingly, I have tried to avoid in this book the pursuit of results not absolutely necessary for the development of the main aspects of the theory, even though such results are quite properly pursued as a matter of course in the more leisurely, comprehensive treatments found in the standard textbooks. At the same time, the restricted treatment given in this book should, by virtue of its conciseness, contribute significantly to the student's understanding and appreciation of the more elaborate developments of the standard texts. With these points in mind, I feel that this book would be suitable for any of the following uses:

(i) A textbook for a half-semester course introducing quantum mechanics to students at the junior or senior level.

(ii) A "warm-up" text or "auxiliary" text for a standard, introductory quantum mechanics course at the senior or beginning graduate level.

(iii) A text for an outside reading course for honors undergraduates.

(iv) A self-study book for students who are now taking, or who have just taken, a standard first course in quantum mechanics.

The book is meant to be completely self-contained, given the following prerequisites: a good background in "sophomore physics" (specifically, an understanding of elementary classical mechanics, plus an awareness of the inadequacy of the classical theory, as exemplified by blackbody radiation, the Bohr atom, and the Davisson-Germer experiment); a fair understanding of elementary calculus, through the improper integral and integration-by-parts; a good grasp of and feeling for the algebra of vectors in three dimensions, through the dot product; and finally, at least a nodding acquaintance with the notion of a complex number.

Because the aim and approach of this book are quite different from the standard texts on elementary quantum mechanics, a separate "Teacher's Preface" follows these remarks. There I have tried to explain very briefly the plan and organization of the book for the benefit of those who already have some knowledge of the subject.

I am happy to acknowledge the help of a number of people who have in various ways contributed to the making of this book. First of all, I owe a debt of gratitude to Professor F. Mandl, through whose extraordinarily well-written book *Quantum Mechanics* [Butterworths Publications, Ltd., London] I first learned much of what I know about this subject. The general approach of my book follows in many respects the approach of Mandl's book, and if my book can accomplish on its level something of what Mandl's does on a more advanced level, then I will be quite pleased. I want to thank my 1966–67 General Physics class at Johns Hopkins University for so willingly submitting themselves to that novel experiment; without their genuine interest in and reaction to my original lectures, this book surely could not have been written. It is a particular pleasure to thank Professor Brian R. Judd of Johns Hopkins University, not only for his graciously consenting to read two drafts of the manuscript, but also for his advice and encouragement. I am very grateful to my good friends Professor Joseph D. Sneed of Stanford University, Dr. Gary A. Prinz of NRL, Professor James S. Marsh of the University of West Florida, Dr. Edward J. Moses of Vanderbilt University, and Professor Ed S. Dorman of Western Kentucky University, for giving me their carefully considered opinions on various portions of the book. In thanking all the aforementioned individuals, I do not want to give the impression that my book necessarily mirrors in every detail their own respective views of quantum mechanics; indeed, the fact that this subject often means such different things to different people is to me one of its most fascinating and enjoyable attributes.

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April, 1970

Teacher's Preface

The general plan of the book is fairly well conveyed through the Table of Contents: Chapter 1 orients the student; Chapter 2 develops the required mathematics; Chapter 3 reviews briefly the salient features of elementary classical mechanics (and in the process gives a simplified derivation and discussion of Hamilton's equations); and the remaining three-quarters of the book is given over to Chapter 4 for the development of quantum mechanics.

The quantum theory is developed for a nonrelativistic system with one degree of freedom via the position representation of the Schrödinger picture—i.e., wave mechanics in one dimension. However, every attempt is made to do this under the guise of the general theory of Dirac; more specifically, even though the Dirac "bra-ket" notation is *not* used, the central theme of the development is that quantum mechanics deals, on the mathematical level, with vectors and operators in a Hilbert space. Accordingly, $\Psi_t(x)$ is consistently referred to as the state *vector* of the system, which can be "expanded as a linear combination of the eigenvectors $\{\alpha_n(x)\}$ of the observable operator $\hat{A}$, $\Psi_t(x) = \Sigma_n (\alpha_n, \Psi_t)\alpha_n(x)$, inasmuch as these eigenvectors form an orthonormal basis in the infinite-dimensional Hilbert space." Such an approach should convey to the student a much deeper understanding of the theory than would be conveyed by a straight "wave-function" approach, but it clearly requires a simplified yet meaningful exposition of the mathematics of the Hilbert space. This is given in Secs. 2-2, 2-3, and 2-4. The key to this exposition is in Sec. 2-3, where the concept of a Hilbert space vector is developed in close analogy with the familiar notion of a vector in three dimensions. The import of this section is summarized in the table on page 23, and warrants further comment.

The main vector analogy is actually drawn between a directed line segment **v**, and a complex function ψ of a real variable x; it is *not* drawn, at least initially, between their n-tuple components $\{v_i\}$ and

$\{\psi(x)\}$. In other words, in Sec. 2-3 we do not regard the x in $\psi(x)$ as a continuous component index, but rather as merely the argument of the function ψ, which itself is the abstract Hilbert space vector. This is done in order to emphasize the important fact that a vector in Hilbert space exists independently of any particular basis representation, even as does a vector in three dimensions. By drawing an analogy between the inner-product *definitions*, $\mathbf{v}_1 \cdot \mathbf{v}_2 \equiv v_1 v_2 \cos\theta_{12}$ and $(\psi_1, \psi_2) \equiv \int \psi_1^*(x)\psi_2(x)dx$, which are viewed merely as ways of deriving a scalar from two vectors such that certain rules are obeyed, one can then pass to any particular n-tuple representation of the vectors $\mathbf{v}$ and $\psi(x)$ through the sets of inner products $\{\mathbf{e}_n \cdot \mathbf{v}\}$ and $\{(\epsilon_n, \psi)\}$. After the Dirac delta function is discussed in Sec. 4-6b, it is shown that a Hilbert space vector "represents itself" with respect to the eigenbasis vectors of the position operator—i.e., that $\{(\delta_\nu, \psi)\} = \{\psi(\nu)\}$. But this fact is regarded as merely an "interesting result," and not as a key feature of the Hilbert space concept. This point of view is not only consistent, but has the important pedagogical advantage of removing the conceptual narrowness of working in the position representation, thereby bringing the student much closer to the real spirit of Dirac's coordinate-free approach.

In order to keep the development of the quantum theory as uncomplicated as possible, three simplifying restrictions are imposed: (i) the system has only one degree of freedom, (ii) operator eigenvalues are entirely discretely distributed, and (iii) operator eigenvalues are nondegenerate. However, in the final sections of the book (Secs. 4-6a, 4-6b, and 4-6c), brief discussions are given showing how the theory is modified when each of these three restrictions is removed. The Dirac delta function in particular is sidestepped as much as possible until Sec. 4-6b, and even there is discussed only briefly.

The method of presentation in Chapter 4 is postulative-deductive: the game is to derive, develop and synthesize the implications of six fundamental postulates into a coherent picture of quantum mechanics. These postulates are found on the following pages:

Postulate 1 (the state vector) . . . . p. 41
Postulate 2 (observable operators) . . . . p. 43
Postulate 3 (measurement predictions). . . . . p. 49
Postulate 4 (measurement effects) . . . . p. 58
Postulate 5 (time evolution). . . . . p. 71
Postulate 6 (position and momentum) . . . . . p. 86

Throughout the development it is emphasized that the chief difference between classical and quantum mechanics lies not in their respective schemes for the time evolution of a system, but rather in their respective concepts of "state," "observables," and "measurement." The first three sections of Chapter 4, encompassing Postulates 1 through 4, are devoted to a careful elucidation of these concepts. The treatment of these matters adheres to the orthodox "Copenhagen" interpretation of quantum mechanics, inasmuch as it seems to have won the acceptance of most physicists today. The essential thesis of the Copenhagen interpretation is that quantum mechanics provides a complete and objective description of the dynamics of a single system, and in particular that it is *not* just a description of our state of knowledge of these dynamics, *nor* is it *merely* a description of the average dynamics of a statistical ensemble of systems. The Copenhagen interpretation implies some rather radical conclusions with regard to the concepts of "state," "observables," and "measurement," and by and large these conclusions are developed openly and taken seriously (as in, for example, the discussion in Sec. 4-3b of the "value of an observable," and the discussion in Sec. 4-5b of the "wave-particle duality"). In adhering to the Copenhagen interpretation, it is not intended to imply that this interpretation is necessarily better or more correct than any other; but the fact is that if one wishes to "understand" quantum mechanics at all, then one cannot avoid adopting, if only tentatively, *some* particular point of view, and it seems reasonable at this stage to adopt the most commonly accepted one.

The discussion of the theory of measurement in Sec. 4-3 is based on a rather idealistic or simplistic definition of a measurement —that it is an in-principle well-defined physical operation, which, when performed on a system, yields a single, errorless, real number. The concepts of the expectation value and uncertainty are developed with special care, the groundwork for this treatment having been laid earlier in Sec. 2-1 by discussing a simple "random drawing" experiment. A careful discussion of the problem of compatibility is given, which culminates in stating and proving the Compatibility Theorem and the Heisenberg Uncertainty Principle.

In Sec. 4-4, the general problem of the time evolution of the quantum state is taken up (Postulate 5). Emphasis is laid on the similarities between the time evolutions of the quantum and classical states. Also stressed is the intimate connection between the energy of a system and its temporal development, as is exemplified by: the time-evolution operator, the time-energy uncertainty principle, the requirement for an observable to be a constant of the motion, and the important role of the stationary states. This book does not take the approach, as some do, of identifying the energy operator with the operator $i\hbar\partial/\partial t$; it is felt that this is inconsistent in a nonrelativistic theory, where the Hilbert space vectors are not functions of time.

The introduction of the *specific* observable operators $\hat{X} = x$ and $\hat{P} = -i\hbar\partial/\partial x$ is finally made in Sec. 4-5 (Postulate 6). Once these two operators are introduced, the two Schrödinger equations, the position-momentum uncertainty relation, the concepts of position probability density and position probability current, and the Ehrenfest equations, are all *derived* from the more general formulation of the preceding four sections. Also prominently featured in this section are a discussion of the wave-particle duality, a discussion of the classical limit of quantum mechanics, and a treatment of the infinite-square well (the only "real problem" that is considered in the entire book).

The book concludes with Sec. 4-6, which attempts to give the student a rough idea of what happens when the previously mentioned simplifying restrictions are removed. This section is intended chiefly to smooth the way to the more detailed and comprehensive treatments of the standard texts.

A total of 73 exercises are closely interwoven with the text. Most of these exercises are short and easy; they are mainly intended to give the student a greater sense of active participation in the development of the theory.

The concise formulation of the Compatibility Theorem is taken from the book of F. Mandl,† which also inspired the analogy between $\mathcal{E}_3$ and $\mathcal{H}$, as well as the overall approach of the fundamental postulates. The development of the time-energy uncertainty relation and the discussion of the classical interpretation of Ehrenfest's equations follow the treatments given in the text of A. Messiah.‡ The author gratefully acknowledges the special help received from these two "standard" textbooks, and commends both to the reader who is interested in pursuing quantum mechanics beyond the confines of this book.

†F. Mandl, *Quantum Mechanics*, Butterworths Publications, Ltd., London, 1957.

‡Albert Messiah, *Quantum Mechanics* (2 vols.), North-Holland Publishing Co., Amsterdam, 1962.

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