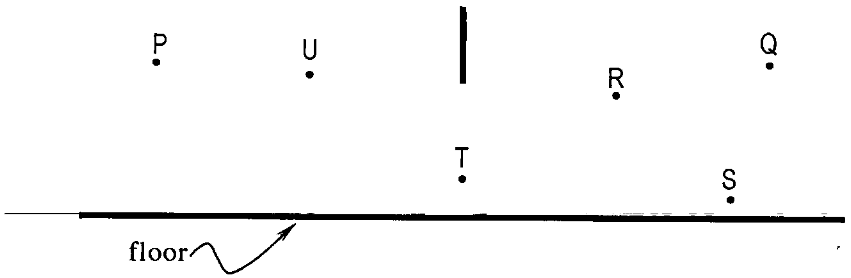
15

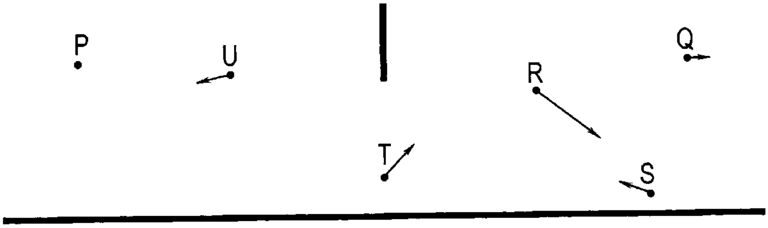
The Wavefunction

15.1 Between release and detection

In the previous chapter, we talked about finding the probability that a ball released at point P would be detected at point Q. We found out how to calculate this probability by assigning an appropriate amplitude arrow to each of the possible paths from P to Q, and then adding up all the arrows. But, what happens if the ball is released at point P and then detected at some other point, say R? (See the figure below.) You know the procedure for finding this probability: enumerate paths from P to R, assign to each path an amplitude arrow using the formula on page 104, and add up all the arrows. It is somewhat more difficult to execute this procedure for the P to R case than it was for the P to Q case, because it lacks the symmetry. Nevertheless it is clear that many of the same features will apply to both processes: for example, in both cases the largest contribution to the sum amplitude arrow comes from a bundle of paths near the path of minimum length. You might find this problem technically difficult, but it is conceptually straightforward and you could do it if you had to.



But we don't have to stop here. We could consider having one detector at Q and another detector at R at the same time. Indeed, we could sprinkle detectors all over the page, at points S, T, U, etc. You know how to find an amplitude arrow for the motion from point P to any of these points, and from the arrow you know how to find the transition probability. The figure below shows what these amplitude arrows might look like.



Now, what if the ball is released at point P, we wait four seconds, and *none* of our detectors go off? How are we to describe the state of the ball after it has been released but not yet detected? We can't say "It's at point R" or "It's at point T" because we don't, indeed we *can't*, know what its position is — the ball doesn't have a position. There is only one way to specify the quantal state of the ball between release and detection, and that is by listing the amplitude arrows for all the points where the ball has some amplitude for being, just as in the figure above. This list is called "the wavefunction".

> *Technical aside:* A word concerning etymology is in order here. In mathematics, the word "function" means a set of numbers assigned to every point in space, or to every instant of time, or both. For example, if there are waves on the surface of a pond, then the height of water in the pond is a function of both position and time. As we saw in chapter 8, "Optical interference", a set of arrows very much like amplitude arrows can be related to waves like those on a pond. In the early days of quantum mechanics, this analogy was believed to be much stronger than it actually is, so the list of amplitude arrows was named the "wave function". In recognition of the important differences that we now recognize between classical waves and amplitude arrows, today the two words are usually closed up as "wavefunction".

Above we supposed that the ball was released from point P and not detected for four seconds. What would happen if, at the five-second mark, the ball were detected at point T? How do we describe the ball's state the instant after it is detected? The answer is simple: we just say that it is located at point T. We no longer need to keep track of the amplitude arrows at points Q, R, S, etc., because although the ball could have gone to any of them, it didn't. Thus immediately before detection the state of the ball is specified by a bunch of arrows spread over many points, while immediately after detection it is specified by giving just one point. What happened to all those arrows? Nothing happened to them, because they never were there. They were never anything more than mathematical tools to help keep our calculations straight. The process described above is called "the collapse of the wavefunction", and it greatly worries those who think that the amplitude arrows are somehow physically out in space, in the same way that air molecules are physically out in space. You don't have that misimpression, so the collapse shouldn't bother you at all.

15.2 What does an electron look like?

The literal answer to this question is "It doesn't look like anything. An electron is too small to be seen." This answer is in fact the dominant one found in discussions on quantum mechanics. We are told not to ask questions that cannot be answered through direct experiment.* For example if an electron is released at point P and detected at point Q, and moves between the points in total darkness so that it is not possible, even in principle, to determine which route it took in moving from P to Q, then we are told that is is not proper to ask which route it took.

This dominant answer is correct but, in my experience, unsatisfactory. When we ask "What does an electron look like?" we really mean "What is the character (or nature) of an electron?" or "How does an electron behave?" or "How can an electron be visualized?". Humans are visual animals, and even if we are told not to visualize a phenomenon we do so anyway — the pictures just pop into our minds unbidden. In quantum mechanics this often leads to naive and incorrect visualizations, which people continue to carry in their minds precisely because the dominant position encourages them not to critically examine their visualizations. So rather than just ignore the issue I like to face it head on, acknowledging that our classical minds are unlikely to produce perfectly accurate visualizations, but realizing that an imperfect visualization, with its imperfections understood, is far superior to an imperfect visualization which is held uncritically. To paraphrase Socrates, "the unexamined visualization is not worth visualizing".

Let us return to the electron moving from P to Q in complete darkness.†

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[*] For example: "The single electron *does* interfere with itself. But don't try to visualize how it does so!"

[†] In technical terms, this paragraph and the next point out the difficulty of visualizing the quantal wavefunction in view of the facts that (i) the wavefunction is complex valued and (ii) it exists in

At the instants of its release and its detection the electron behaves like a very small, very hard marble, in that it has a definite position. But between these two events the electron doesn't have a definite position. Sometimes I visualize it as a cloud that is thicker at places where the electron is more likely to be and thinner at places where it is less likely to be. This visualization captures beautifully the probabilistic character of quantum mechanics, but it shows nothing of the interference character. So I sometimes visualize an electron instead through a swarm of rotating amplitude arrows, the swarm being thicker and the arrows longer where the electron is more likely to be. This can give me nightmares, so more often I simply modify the cloud visualization by assigning colors to different arrow directions and mentally coloring each point of the cloud according to the direction of the amplitude arrow there. In my mind's eye, I see the electron as a swirl of shimmering colors. Both of these visualizations can be useful, but both have the defect of infusing a mathematical tool — the amplitude arrow — with physical reality.

The problem becomes even more acute when one attempts to visualize a system of two particles because then (see section 11.2) one must visualize not one state for one particle and another state for the other particle, but instead a single state for the pair of particles.

It is easier to show why some visualizations are poor than to produce visualizations that are good. For example, some people like to visualize an electron as a small hard marble that takes a definite and well-defined route from P to Q, but that the actual route to be taken is not predictable beforehand, so that sometimes the marble will take one route and sometimes it will take another. It is impossible, however, to make such a picture consistent with the interference results of chapter 9. (Or at least, it is impossible to do so without invoking mysterious messages that allow a marble passing through branch a to know whether or not branch b is open or blocked.) So you may not know what an electron looks like, but at least you know what it doesn't look like!

The problem of visualization is closely connected to a problem of terminology. To many, the word "particle" conjures up the image of a small, hard, classical marble. In quantum mechanics, it is not entirely clear what the image associated with "particle" ought to be, but it most certainly is *not* this classical picture! If we were eminent Victorians we would find a noble Latin root and build a new word to describe the quantal particle. If we lived in Washington, DC, or Arlington, Virginia (the site of the Pentagon), we would invent an acronym (something like PAWBITQMF — particle and/or wave behaving in typical quantum mechanical fashion).

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configuration space *or* in momentum space but *not* in ordinary three-dimensional position space.

There have been some attempts to coin a new word: "wavicle", "quon", or "quanton". These attempts have not caught on.

In conclusion, I do not have a visualization — or even a name — that is satisfactory for even so simple a thing as a single quantal electron. Because my mind is filled with classical images and intuition, this is perhaps not surprising. A truly successful visualization would be very close to a classical "clockwork" mechanism that underlies quantum mechanics, and we have already seen (section 9.8) that such a mechanism does not exist. But this lack of visualization must be regarded as a limitation of my imagination, and not as any defect in nature or in quantum mechanics.

> *Our imagination is stretched to the utmost, not, as in fiction, to imagine things which are not really there, but just to comprehend those things which* are *there.*
>
> — Richard Feynman

15.3 Problems

15.1 *Mistaken visualization.* What is wrong with the statement "Between release and detection, the electron might be at any one of many points"? Can you rephrase the statement to make it correct?

15.2 *Wording.* On 28 May 1996 the New York *Times* published an article titled "Team of physicists proves atom can exist in two places at once". The article describes an experiment in which Chris Monroe and coworkers "succeeded in separating two states of a single atom in space, then pulled them 83 nanometers apart". This article's title is perfectly appropriate for an audience unfamiliar with quantum mechanics and its terminology. Now that you do know the terminology of quantum mechanics, think up a more accurate title.

15.3 *Visualization.* On page 176 of his book *In Search of Schrödinger's Cat*, John Gribbin claims that electron interference raises "the puzzle that an electron at hole A *knows* whether hole B is open or closed". Which incorrect visualization of an electron is Gribbin using that makes this phenomenon seem puzzling to him?

15.4 *Need for visualization.* Does our inability to find a satisfactory visualization for a quantal particle mean that the dominant position ("don't ask questions that you can't answer") is the best one after all? Is its absence merely distressing or does it constitute a fundamental flaw in our knowledge? (Let me point out that distressing things are, by definition, not pleasant, but neither, unfortunately, are they rare.)

15.5 *Measurement.* Mr. Parker finds the quantum measurement process difficult to understand. "Suppose I start with an atom in a state so that it has equal probability of being anywhere in a box. If I shine a strong light throughout the entire box I will find the atom only at one point. But what happens if I shine the light on only the left half of the box, and don't find the atom? I now know that the atom is somewhere in the right half. How could the light, shining where the atom isn't, affect the atom?" Convince Mr. Parker that the conflict is not between quantum mechanics and reality, but between quantum mechanics and his incorrect visualization of the atom as a tiny marble. (This conundrum is called the Renninger negative-result experiment.)

15.6 *Visualization techniques.* (For technical readers.) This chapter mentioned two techniques for visualizing wavefunctions: through a swarm of amplitude arrows ("phasors") and through color. I have written a computer program that displays one-dimensional time-varying wavefunctions using either of these techniques, and two other techniques as well. Download the program (it works under the MS-DOS operating system) through the World Wide Web site mentioned on page xiv, and evaluate these different display styles. Can you come up with new visualization techniques of your own? If so, please tell me what they are!

15.7 *Faster-than-light propagation.* (For technical readers.) In a one-particle situation in quantum mechanics, the wavefunction at a given point changes instantly as soon as the particle is detected. In the Coulomb gauge, the electric potential (and the vector potential) at a given point changes the instant that any charged particle, anywhere in the universe, is moved. Does either mechanism permit instantaneous communication?

Appendix A

A Brief History of Quantum Mechanics

Up to now this book has focused on the behavior of nature. I could say more: more about measurement, more about the classical limit, more about different rules for assigning amplitudes, and so forth, but the main points have been made. So instead of talking more about nature I'm going to talk about people — about how people discovered quantum mechanics.

A.1 Warnings

I am not a historian of science. The history of science is a very difficult field. A historian of science must be just as proficient at science as a scientist is, but must also have a good understanding of personalities, and a good knowledge of the social and political background that affects developments in science and that is in turn affected by those developments. He or she has to know not only the outcome of the historical process, namely the science that is generally accepted today, but also the many false turns and blind alleys that scientists tripped across in the process of discovering what we believe today. He or she must understand not only the cleanest and most direct experimental evidence supporting our current theories (like the evidence presented in this book), but must understand also how those theories came to be accepted through a tightly interconnected web of many experiments, no one of which was completely convincing but which taken together presented an overwhelming argument.

Thus a full history of quantum mechanics would have to discuss Schrödinger's many mistresses, Ehrenfest's suicide, and Heisenberg's involvement with Nazism. It would have to treat the First World War's effect on the development of science. It would need to mention "the Thomson model" of the atom, which was once the major competing theory to quantum mechanics. It would have to give appropriate weight to both theoretical and experimental developments. Needless to say, such a complete history will never be written, and this brief appendix will not even broach most of these topics. The references on page 131 will lead you to further information.

The historian of science has problems beyond even these. The work of government is generally carried out through the exchange of written memos, and when verbal arguments are used (as in Congressional hearings) detailed written transcripts are maintained. These records are stored in archives to insure that historians interested in government decisions will have access to them. Historians of science do not have such advantages. Much of the work of science is done through informal conversations, and the resulting written record is often sanitized to avoid offending competing scientists. The invaluable oral record is passed down from professor to student repeatedly before anyone ever records it on paper. Naturally, the stories tend to become better and better as they are transmitted over and over. In addition, there is a tendency for the exciting stories to be repeated and the dull ones to be forgotten, leading to a Darwinian "survival of the funniest" — rather than of the most accurate.

Finally, once all the historical records have been sifted and analyzed, there remains the problem of overall synthesis and presentation. Many scientific historians (and even more scientists) like to tell a story in which each step follows naturally from the one preceding it, scientists always work cooperatively and selflessly, and where harmony rules.* Such stories infuriate me. They remind me of the stock market analysts who come onto television every evening and explain in detail the cause of every dip and curve in the Dow for the preceding day. If they know the stock market so well, why do they wait until evening to tell me about it? Why don't they tell me in the morning so that it can do me some good? For that matter, why are they on television at all, rather than out relaxing on their million-dollar yachts? The fact is that scientific history, like the stock market and like everyday life, does not proceed in an orderly, coherent pattern. The story of quantum mechanics is a story full of serendipity, personal squabbles, opportunities missed and taken, and of luck both good and bad.

Because I find the sugar-sweet stories of the harmonious development of science to be so offensive, when I tell the story I emphasize the conflicts, the contingencies, and the unpredictablities. Hence the story I tell is no more accurate than the sweet talk, because I go too far in the opposite direction. Keep in mind, as you read the story that follows, that I suffer

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* I told a story like this myself in section 10.2, "Evidence for the amplitude framework", where I suggested that discoveries in physics always result from the exploration of shorter length scales. In fact, discoveries also come from the exploration of longer length scales, of lower temperatures, of greater complexity, and simply by investigating familiar phenomena in more detail.

from this overreaction as well as all the other difficulties mentioned in this section.

A.2 Status of physics: January 1900

In January 1900 the atomic hypothesis was widely but not universally accepted. Atoms were considered point particles, and it wasn't clear how atoms of different elements differed. The electron had just been discovered (1897) and it wasn't clear where (or even whether) electrons were located within atoms. One important outstanding problem concerned the colors emitted by atoms in a discharge tube (familiar today as the light from a fluorescent tube or from a neon sign). No one could understand why different gas atoms glowed in different colors. Another outstanding problem concerned the amount of heat required to change the temperature of a diatomic gas such as oxygen: the measured amounts were well below the value predicted by theory. Because quantum mechanics is important when applied to atomic phenomena, you might guess that investigations into questions like these would give rise to the discovery of quantum mechanics. Instead it came from a study of heat radiation.

A.3 Heat radiation

You know that the coals of a campfire, or the coils of an electric stove, glow red. You probably don't know that even hotter objects glow white, but this fact is well known to blacksmiths. When objects are hotter still they glow blue. (This is why a gas stove should be adjusted to make a blue flame.) Indeed, objects at room temperature also glow (radiate), but the radiation they emit is infrared, which is not detectable by the eye. (The military has developed — for use in night warfare — special eye sets that convert infrared radiation to optical radiation.)

These observations can be explained qualitatively by thinking of heat as a jiggling of atoms: like jello, but on a smaller scale so that you can't see the vibrations due to heat. At higher temperatures the atoms jiggle both farther and faster. The increased distance of jiggling accounts for the brighter radiation from hotter bodies, while the increased speed accounts for the change in color.

In the year 1900 several scientists were trying to turn these observations into a detailed explanation of and a quantitatively accurate formula for the color of heat radiation as a function of temperature. On 19 October 1900 the Berliner Max Planck (age 42) announced a formula that fit the experimental results perfectly, yet he had no explanation for the formula — it just happened to fit. He worked to find an explanation through the late fall and finally was able to derive his formula by assuming that the atomic jigglers could not take on any possible energy, but only certain special "allowed" values. He announced this result on 14 December 1900. This date is now considered the birthday of quantum mechanics (and there is certain to be a big celebration on its one hundredth anniversary) but at the time no one found it particularly significant. We know this not only from contemporary reports, but also because the assumption of allowed energy values raises certain obvious questions that no one bothered to follow up. For example, how does the jiggler change from one allowed energy to another if the intermediate energies are prohibited? Again, if a jiggling atom can only assume certain allowed values of energy, then there must also be restrictions on the positions and speeds that the atom can have. What are they? Planck never tried to find out.

Thirty-one years after his discovery Planck wrote:

> I can characterize the whole procedure as an act of desperation, since, by nature I am peaceable and opposed to doubtful adventures. However, I had already fought for six years (since 1894) with the problem of equilibrium between radiation and matter without arriving at any successful result. I was aware that this problem was of fundamental importance in physics, and I knew the formula describing the energy distribution ... hence a theoretical interpretation *had* to be found at any price, however high it might be.

It should be clear from what I have already said that this is just a beautiful and romantic story that was developed with good thirty-year hindsight. Here is another wonderful story, this one related by Werner Heisenberg:

> In a period of most intensive work during the summer of 1900 [Planck] finally convinced himself that there was no way of escaping from this conclusion [of "allowed" energies]. It was told by Planck's son that his father spoke to him about his new ideas on a long walk through the Grunewald, the wood in the suburbs of Berlin. On this walk he explained that he felt he had possibly made a discovery of the first rank, comparable perhaps only to the discoveries of Newton.

As much as I would like for this beautiful story to be true, the intensive work took place during the late fall, not the summer, of 1900. If Planck did indeed take his son for a long walk on the afternoon that he discovered quantum mechanics, the son would probably remember the nasty cold he caught better than any remarks his father made.

A.4 The old quantum theory

Although the ideas of Planck did not take the world by storm, they did develop a growing following and were applied to more and more situations. The resulting ideas, now called "old quantum theory", were all of the same type: Classical mechanics was assumed to hold, but with the additional assumption that only certain values of a physical quantity (the energy, say, or the projection of a magnetic arrow) were allowed. Any such quantity was said to be "quantized". The trick seemed to be to guess the right quantization rules for the situation under study, or to find a general set of quantization rules that would work for all situations.

For example, in 1905 Albert Einstein (age 26) postulated that the total energy of a beam of light is quantized. Just one year later he used quantization ideas to explain the heat/temperature puzzle for diatomic gases. Five years after that, in 1911, Arnold Sommerfeld (age 43) at Munich began working on the implications of energy quantization for position and speed.

In the same year Ernest Rutherford (age 40), a New Zealander doing experiments in Manchester, England, discovered the atomic nucleus — only at this relatively late stage in the development of quantum mechanics did physicists have even a qualitatively correct picture of the atom! In 1913, Niels Bohr (age 28), a Dane who had recently worked in Rutherford's laboratory, introduced quantization ideas for the hydrogen atom. His theory was remarkably successful in explaining the colors emitted by hydrogen glowing in a discharge tube, and it sparked enormous interest in developing and extending the old quantum theory.

This development was hindered but not halted completely by the start of the First World War in 1914. During the war (in 1915) William Wilson (age 40, a native of Cumberland, England, working at King's College in London) made progress on the implications of energy quantization for position and speed, and Sommerfeld also continued his work in that direction.

With the coming of the armistice in 1918, work in quantum mechanics expanded rapidly. Many theories were suggested and many experiments performed. To cite just one example, in 1922 Otto Stern and his graduate student Walther Gerlach (ages 34 and 23) performed their important experiment that is so essential to the way this book presents quantum mechanics. Jagdish Mehra and Helmut Rechenberg, in their monumental history of quantum mechanics, describe the situation at this juncture well:

> At the turn of the year from 1922 to 1923, the physicists looked forward with enormous enthusiasm towards detailed solutions of the outstanding problems, such as the helium problem and

the problem of the anomalous Zeeman effects. However, within less than a year, the investigation of these problems revealed an almost complete failure of Bohr's atomic theory.

A.5 The matrix formulation of quantum mechanics

As more and more situations were encountered, more and more recipes for allowed values were required. This development took place mostly at Niels Bohr's Institute for Theoretical Physics in Copenhagen, and at the University of Göttingen in northern Germany. The most important actors at Göttingen were Max Born (age 43, an established professor) and Werner Heisenberg (age 23, a freshly minted Ph.D. from Sommerfeld in Munich). According to Born "At Göttingen we also took part in the attempts to distill the unknown mechanics of the atom out of the experimental results. ... The art of guessing correct formulas ... was brought to considerable perfection."

Heisenberg particularly was interested in general methods for making guesses. He began to develop systematic tables of allowed physical quantities, be they energies, or positions, or speeds. Born looked at these tables and saw that they could be interpreted as mathematical matrices. Fifty years later matrix mathematics would be taught even in high schools. But in 1925 it was an advanced and abstract technique, and Heisenberg struggled with it. His work was cut short in June 1925. As Mehra and Rechenberg describe it:

> This was late spring in Göttingen, with fresh grass and flowering bushes, and Heisenberg was interrupted in his work by a severe attack of hay fever. Since he could hardly do anything, he had to ask his director, Max Born, for a leave of about two weeks, which he decided to spend on the rocky island of Helgoland to effect a cure.
>
> On 7 June 1925 Heisenberg took the night train from Göttingen to Cuxhaven where he had to catch the ferryboat for Helgoland in the morning. On arrival at Cuxhaven, "I was extremely tired and my face was swollen. I went to get breakfast in a small inn and the landlady said, 'You must have had a pretty bad night. Somebody must have beaten you.' She thought I had had a fight with somebody. I told her that I was ill and that I had to take the boat, but she was still worried about me." A few hours later he reached Helgoland.
>
> Helgoland, a rocky island in the North Sea, consists of a mass of red sandstone, rising abruptly to an elevation of about 160 feet, and there is nearly no vegetation on it. [It has an

> area of about 380 acres and a permanent population of several hundred inhabitants. On the lower section of the island lies a fishing village, while the upper section serves as a summer resort for tourists. ... From 1402 to 1714 it formed a part of Schleswig-Holstein, then became Danish until it was seized by the English fleet in 1807. It was formally ceded to Great Britain in 1814. Britain gave it to Germany in exchange for Zanzibar and some territory in Africa (1890). Helgoland was an important base for the German Navy. In accordance with the Treaty of Versailles the military and naval fortifications were demolished in 1920–1922. Under the Nazi régime Helgoland again became a military stronghold and was a target for heavy Allied bombing towards the end of World War II. From 1947 to 1 March 1952, when it was handed back to Germany, the island was used as a bombing range by the Royal Air Force. Then it was restored as a tourist and fishing center.] Heisenberg rented a room on the second floor of a house situated high above the southern edge of the island, which offered him a "glorious view over the village, and the dunes and the sea beyond." "As I sat on my balcony," he recalled more than forty years later, "I had ample opportunity to reflect on Bohr's remark that part of infinity seems to lie within the grasp of those who look across the sea." He began to take walks to the upper end of the island and swam daily in the sea. Soon he felt much better, and he began to divide his time into three parts. The first he still used for walking and swimming; the second he spent in reading Goethe's *West-Östlicher Divan*; and the third he devoted to work on physics. Having nothing else to distract him, he could reflect with great concentration on the problems and difficulties which had been occupying him until a few days earlier in Göttingen.

Heisenberg reproduced his earlier work, cleaning up the mathematics and simplifying the formulation. He worried that the mathematical scheme he invented might prove to be inconsistent, and in particular that it might violate the principle of the conservation of energy. In Heisenberg's own words:

> One evening I reached the point where I was ready to determine the individual terms in the energy table, or, as we put it today, in the energy matrix, by what would now be considered an extremely clumsy series of calculations. When the first terms seemed to accord with the energy principle, I became rather excited, and I began to make countless arithmetical errors. As

> a result, it was almost three o'clock in the morning before the final result of my computations lay before me. The energy principle had held for all the terms, and I could no longer doubt the mathematical consistency and coherence of the kind of quantum mechanics to which my calculations pointed. At first, I was deeply alarmed. I had the feeling that, through the surface of atomic phenomena, I was looking at a strangely beautiful interior, and felt almost giddy at the thought that I now had to probe this wealth of mathematical structures nature had so generously spread out before me. I was far too excited to sleep, and so, as a new day dawned, I made for the southern tip of the island, where I had been longing to climb a rock jutting out into the sea. I now did so without too much trouble, and waited for the sun to rise.

By the end of the summer Heisenberg, Born, and Pascual Jordan (age 22) had developed a complete and consistent theory of quantum mechanics. (Jordan had entered the collaboration when he overheard Born discussing quantum mechanics with a colleague on a train.)

This theory, called "matrix mechanics" or "the matrix formulation of quantum mechanics", is not the theory I have presented in this book. It is extremely and intrinsically mathematical, and even for master mathematicians it was difficult to work with. Although we now know it to be complete and consistent, this wasn't clear until much later. Heisenberg had been keeping Wolfgang Pauli apprised of his progress. (Pauli, age 25, was Heisenberg's friend from graduate student days, when they studied together under Sommerfeld.) Pauli found the work too mathematical for his tastes, and called it "Göttingen's deluge of formal learning". On 12 October 1925 Heisenberg could stand Pauli's biting criticism no longer. He wrote to Pauli:

> With respect to both of your last letters I must preach you a sermon, and beg your pardon for proceeding in Bavarian: It is really a pigsty that you cannot stop indulging in a slanging match. Your eternal reviling of Copenhagen and Göttingen is a shrieking scandal. You will have to allow that, in any case, we are not seeking to ruin physics out of malicious intent. When you reproach us that we are such big donkeys that we have never produced anything new in physics, it may well be true. But then, you are also an equally big jackass because you have not accomplished it either...... (The dots denote a curse of about two-minute duration!) Do not think badly of me and many greetings.

A.6 The wavefunction formulation of quantum mechanics

While this work was going on at Göttingen and Helgoland, others were busy as well. In 1923 Louis de Broglie (age 31), associated an "internal periodic phenomenon" — a wave — with a particle. He was never very precise about just what that meant. (De Broglie is sometimes called "Prince de Broglie" because his family descended from the French nobility. To be strictly correct, however, only his eldest brother could claim the title.)

It fell to Erwin Schrödinger, an Austrian working in Zürich, to build this vague idea into a theory of wave mechanics. He did so during the Christmas season of 1925 (at age 38), at the alpine resort of Arosa, Switzerland, in the company of "an old girlfriend [from] Vienna", while his wife stayed home in Zürich.

In short, just twenty-five years after Planck glimpsed the first sight of a new physics, there was not one, but two competing versions of that new physics! The two versions seemed utterly different and there was an acrimonious debate over which one was correct. In a footnote to a 1926 paper Schrödinger claimed to be "discouraged, if not repelled" by matrix mechanics. Meanwhile, Heisenberg wrote to Pauli (8 June 1926) that

> The more I think of the physical part of the Schrödinger theory, the more detestable I find it. What Schrödinger writes about visualization makes scarcely any sense, in other words I think it is shit. The greatest result of his theory is the calculation of matrix elements.

Fortunately the debate was soon stilled: in 1926 Schrödinger and, independently, Carl Eckert (age 24) of Caltech proved that the two new mechanics, although very different in superficial appearance, were equivalent to each other.† (Pauli also proved this, but never published the result.)

A.7 Applications

With not just one, but two complete formulations of quantum mechanics in hand, the quantum theory grew explosively. It was applied to atoms, molecules, and solids. It solved with ease the problem of helium (see page 123) that had defeated the old quantum theory. It resolved questions concerning the structure of stars, the nature of superconductors, and the properties of magnets. One particularly important contributor was

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† Very much as the process of adding arabic numerals is very different from the process of adding roman numerals, but the two processes nevertheless always give the same result (see problem 8.2).

P.A.M. Dirac, who in 1926 (at age 22) extended the theory to relativistic and field-theoretic situations. Another was Linus Pauling, who in 1931 (at age 30) developed quantum mechanical ideas to explain chemical bonding, which previously had been understood only on empirical grounds. Even today quantum mechanics is being applied to new problems and new situations. It would be impossible to mention all of them. All I can say is that quantum mechanics, strange though it may be, has been tremendously successful.

A.8 The Bohr–Einstein debate

The extraordinary success of quantum mechanics in applications did not overwhelm everyone. A number of scientists, including Schrödinger, de Broglie, and — most prominently — Einstein, remained unhappy with the standard probabilistic interpretation of quantum mechanics. In a letter to Max Born (4 December 1926), Einstein made his famous statement that

> Quantum mechanics is very impressive. But an inner voice tells me that it is not yet the real thing. The theory produces a good deal but hardly brings us closer to the secret of the Old One. I am at all events convinced that *He* does not play dice.

In concrete terms, Einstein's "inner voice" led him, until his death, to issue occasional detailed critiques of quantum mechanics and its probabilistic interpretation. Niels Bohr undertook to reply to these critiques, and the resulting exchange is now called the "Bohr–Einstein debate". At one memorable stage of the debate (Fifth Solvay Congress, 1927), Einstein made an objection similar to the one quoted above and Bohr

> replied by pointing out the great caution, already called for by ancient thinkers, in ascribing attributes to Providence in every-day language.

These two statements are often paraphrased as, Einstein to Bohr: "God does not play dice with the universe." Bohr to Einstein: "Stop telling God how to behave!" While the actual exchange was not quite so dramatic and quick as the paraphrase would have it, there was nevertheless a wonderful rejoinder from what must have been a severely exasperated Bohr.

The Bohr–Einstein debate had the benefit of forcing the creators of quantum mechanics to sharpen their reasoning and face the consequences of their theory in its most starkly non-intuitive situations. It also had (in my opinion) one disastrous consequence: because Einstein phrased his objections in purely classical terms, Bohr was compelled to reply in nearly classical terms, giving the impression that in quantum mechanics, an electron is "really classical" but that somehow nature puts limits on how well we can determine those classical properties. I have tried in this book to convince you that this is a misconception: the reason we cannot measure simultaneously the exact position and speed of an electron is because an electron does *not have* simultaneously an exact position and speed. It is no defect in our measuring instruments that they cannot measure what does not exist. This is simply the character of an electron — an electron is *not* just a smaller, harder edition of a marble. This misconception — this picture of a classical world underlying the quantum world — poisoned my own understanding of quantum mechanics for years. I hope that you will be able to avoid it.

On the other hand, the Bohr–Einstein debate also had at least one salutary product. In 1935 Einstein, in collaboration with Boris Podolsky and Nathan Rosen, invented a situation in which the results of quantum mechanics seemed completely at odds with common sense, a situation in which the measurement of a particle at one location could reveal instantly information about a second particle far away. The three scientists published a paper which claimed that "No reasonable definition of reality could be expected to permit this." Bohr produced a recondite response and the issue was forgotten by most physicists, who were justifiably busy with the applications of rather than the foundations of quantum mechanics. But the ideas did not vanish entirely, and they eventually raised the interest of John Bell. In 1964 Bell used the Einstein–Podolsky–Rosen situation to produce a theorem about the results from certain distant measurements for any deterministic scheme, not just classical mechanics. In 1982 Alain Aspect and his collaborators put Bell's theorem to the test and found that nature did indeed behave in the manner that Einstein (and others!) found so counterintuitive.

A.9 The amplitude formulation of quantum mechanics

The version of quantum mechanics presented in this book is neither matrix nor wave mechanics. It is yet another formulation, different in approach and outlook, but fundamentally equivalent to the two formulations already mentioned. It is called amplitude mechanics (or "the sum over histories technique", or "the many paths approach", or "the path integral formulation", or "the Lagrangian approach", or "the method of least action"), and it was developed by Richard Feynman in 1941 while he was a graduate student (age 23) at Princeton. Its discovery is well described by Feynman himself in his Nobel lecture:

> I went to a beer party in the Nassau Tavern in Princeton. There was a gentleman, newly arrived from Europe (Herbert

> Jehle[‡]) who came and sat next to me. Europeans are much more serious than we are in America because they think a good place to discuss intellectual matters is a beer party. So he sat by me and asked, "What are you doing" and so on, and I said, "I'm drinking beer." Then I realized that he wanted to know what work I was doing and I told him I was struggling with this problem, and I simply turned to him and said "Listen, do you know any way of doing quantum mechanics starting with action — where the action integral comes into the quantum mechanics?" "No," he said, "but Dirac has a paper in which the Lagrangian, at least, comes into quantum mechanics. I will show it to you tomorrow."
>
> Next day we went to the Princeton Library (they have little rooms on the side to discuss things) and he showed me this paper.

Dirac's short paper in the *Physikalische Zeitschrift der Sowjetunion* claimed that a mathematical tool which governs the time development of a quantal system was "analogous" to the classical Lagrangian.

> Professor Jehle showed me this; I read it; he explained it to me, and I said, "What does he mean, they are analogous; what does that mean, *analogous?* What is the use of that?" He said, "You Americans! You always want to find a use for everything!" I said that I thought that Dirac must mean that they were equal. "No," he explained, "he doesn't mean they are equal." "Well," I said, "let's see what happens if we make them equal."
>
> So, I simply put them equal, taking the simplest example ... but soon found that I had to put a constant of proportionality A in, suitably adjusted. When I substituted ... and just calculated things out by Taylor-series expansion, out came the Schrödinger equation. So I turned to Professor Jehle, not really understanding, and said, "Well you see Professor Dirac meant that they were proportional." Professor Jehle's eyes were bugging out — he had taken out a little notebook and was rapidly copying it down from the blackboard and said, "No, no, this is an important discovery."

Feynman's thesis advisor, John Archibald Wheeler (age 30), was equally impressed. He believed that the amplitude formulation of quantum me-

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[‡] Jehle had been a student of Schrödinger in Berlin, and was in Princeton fleeing the Nazis. He was a Quaker and had survived prison camps in both Germany and France.

chanics — although mathematically equivalent to the matrix and wave formulations — was so much more natural than the previous formulations that it had a chance of convincing quantum mechanics's most determined critic. Wheeler writes:

> Visiting Einstein one day, I could not resist telling him about Feynman's new way to express quantum theory. "Feynman has found a beautiful picture to understand the probability amplitude for a dynamical system to go from one specified configuration at one time to another specified configuration at a later time. He treats on a footing of absolute equality every conceivable history that leads from the initial state to the final one, no matter how crazy the motion in between. The contributions of these histories differ not at all in amplitude, only in phase. ... This prescription reproduces all of standard quantum theory. How could one ever want a simpler way to see what quantum theory is all about! Doesn't this marvelous discovery make you willing to accept the quantum theory, Professor Einstein?" He replied in a serious voice, "I still cannot believe that God plays dice. But maybe", he smiled, "I have earned the right to make my mistakes."

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Appendix B

Putting Weirdness to Work

According to Charles de Gaulle, Napoleon's military genius lay in his ability "to grasp the situation, to adapt himself to it, and to exploit it to his own advantage". Most of this book has treated the first two of these steps: learning what quantum mechanics is and how to work with it, whether we like it or not. This appendix moves on to the third step of exploitation.

The applications of quantum mechanics are myriad. Quantum mechanics underlies all chemical and biochemical reactions, the design of drugs and of alloys, and the generation of medical X-rays. It is essential to the laser, to the transistor, and to a sensitive detector of magnetic field called the SQUID (Superconducting QUantum Interference Device). But for the purposes of this book, it is useful to focus on only three of these applications: quantum cryptography, tunneling applications, and quantum computers. The first of these was treated in chapter 13; this appendix describes the second and third. These descriptions are segregated into an appendix because I don't know how to treat them thoroughly at the mathematical level of this book. Consequently, the treatments here are more descriptive and less analytic than the treatments in the chapters.

B.1 Tunneling

A classical ball rolls in a bowl. Can the ball escape? As the ball rolls up the side of the bowl, it slows down. If the ball has enough energy, it will slow down but not stop, and hence can make it over the side and out. A ball with a low enough energy will always remain inside the bowl.

Is there any difference if we use a quantal ball? In this case, as we have seen, the ball might not have a definite position, so there are situations in which it has some amplitude for being inside the bowl and some amplitude for being outside the bowl. It is also true (although we have not demonstrated this) that the ball might not have a definite energy, so there are situations in which the average energy is too small for the ball to escape, but yet there is some amplitude for the ball to have enough energy to escape. Thus it can happen that a quantal ball starts well inside the bowl with an average energy too small for classical escape, yet nevertheless the ball escapes. This process is called tunneling, because it is a way to get out of a barrier without going over the barrier. (The name unfortunately suggests that the quantal ball bores a hole through the side of the bowl. It doesn't — the bowl is unaltered.)

Are there any practical applications for tunneling? Prisoners might hope to tunnel through the walls of their jail cells, but this is not a practical application: the probability of tunneling through a barrier decreases dramatically with the thickness of the barrier. But this same feature that makes tunneling impractical for prison escape is essential for a device that locates atoms. In this device a thin needle moves across the surface of a sample. Electrons can tunnel from the needle to the sample, but only if the two are very close. In this way, a very precise picture of the sample's surface can be build up. This device, called a "scanning tunneling microscope", can easily locate individual atoms.

Tunneling is also important in the decay of atomic nuclei, for an esoteric electronic component called the "tunnel diode", and as a possible mechanism for superconductivity at high temperatures. My favorite application of tunneling, however, is far from recondite.

The sun produces light energy through a series of nuclear reactions. The first step in this series is that two protons come very close to each other and react to form a proton and neutron bound together, plus a positron, plus a neutrino. If you don't know what a positron is, don't worry. The important thing is that the two protons have to come close together. But the two protons have the same electric charge, so they repel each other strongly. Calculations based on classical mechanics predict that this reaction would happen so slowly that almost no light would come from the sun. A correct calculation based on quantum mechanics shows that one proton tunnels through the barrier of repulsion separating the two, and allows the reaction to proceed.

Quantum mechanics applies to the domain of the very small, but sometimes small things have big consequences. Sunshine itself is generated through the workings of quantum mechanics.

B.2 Quantum computers

Not so many years ago, it was customary to interpret the Heisenberg uncertainty principle as a limitation on information: "In classical mechanics one can know a particle's position and its speed exactly, but in quantum mechanics one cannot have this complete information." This is quite the wrong attitude. In fact, one may have complete information concerning either a classical state or a quantal state, but the information is different in the two cases. Consider, for example, a single bead strung on a fixed wire. In classical mechanics, the bead's state is specified by listing its position and its speed: two numbers. In quantum mechanics, the bead's state is specified (see chapter 15, "The wavefunction") by listing the amplitude for it to be at any of the points along the wire. Since there are an infinite number of points on the wire, and since the amplitude at each point is specified through two numbers (a magnitude and an angle), specifying a quantal state actually requires considerably *more* information than does specifying a classical state.

In short, the information needed to specify a quantal state is not only different in character from the information needed to specify a classical state, but it is also much larger in quantity. Thus there are many more quantal states than there are classical states for the same system. This fact is a source of both delight and difficulty. The delight stems from the great richness and variety of quantal behavior, a variety lacking in the classical domain simply because there are many more ways to be quantal than there are ways to be classical. The difficulty lies in the fact that calculations involving quantal systems necessarily process a lot more information than those involving the corresponding classical system, and thus are usually more difficult to perform. A computer program simulating a quantal system will almost always run slower than one simulating the corresponding classical system: the quantum simulation simply has more information to keep track of.

For many years, this was regarded as an unpleasant but unavoidable fact of scientific life. Then, in the 1980s, three scientists (Paul Benioff, Richard Feynman, and David Deutsch) realized that this difficulty could be profitably turned around. Instead of complaining about the problems of simulating quantum mechanics using classical computers, couldn't we build computers out of quantal systems? The richness of quantum mechanics might then allow such "quantum computers" to accomplish more tasks faster than their classical counterparts. For example, in a conventional computer the memory consists of many storage locations that can be set to either "1" or "0", and the processor consists of many switches that can be either "up" or "down". But a quantal system — such as the magnetic needle of a silver atom — can be either "up" ($m_z = +m_B$), or "down" ($m_z = -m_B$), or in an infinite number of other possibilities. Pieces of a quantum computer can interfere or become entangled, options that are not available to the components of classical computers. Can this flexibility be harnessed to make quantal storage locations or switches that work harder than their classical counterparts?

The answer to this question is "yes". For example, in 1997 Lov Grover showed how a quantum computer could outperform a classical computer in searching through an unordered list. Suppose, for instance, that you wanted some information and you knew it was contained in one of ten million possible World Wide Web sites. If a computer could examine one Web site per second, then a classical computer would need on average five million seconds — two months — to find the desired site. A similar quantum computer would find it in forty-two minutes. In 1998 Chuang, Gershenfeld, and Kubinec built a quantum computer that implemented Grover's idea, but the computer could not search through a list of ten million possibilities: it was restricted to lists of four items.

Many issues, both fundamental and technical, must be resolved before the quantum computer becomes more than a laboratory curiosity. Quantum computers may lead society into an information revolution that will make the classical computer revolution look like a ripple. Or the whole idea might just fizzle. But in either case quantum computing illustrates that the quantal domain is fundamentally different from the classical domain, offering up a set of possibilities so various, so beautiful, so new, that they demand a fresh picture of this extraordinary universe, our home.

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Appendix C

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Appendix D

General Questions

Many chapters in this book are followed by problems (see page 10) that pertain specifically to that chapter. This appendix contains questions of a more general character. These questions are designed either to consolidate your understanding or to extend your knowledge. The latter sort of question will require further study, such as through reading books listed in the references. But answering the questions will generally require considerable analytic thought and not just parroting a book from the library.

D.1 *Is God a deceiver?* A central element of René Descartes's philosophy is that we can usually trust our sensual perceptions because God is not a deceiver. The macroscopic world seems to obey the deterministic laws of Newton, yet quantum mechanics maintains that this is just an appearance: the actual laws of physics are probabilistic not deterministic. Does this mean that Descartes was wrong and that God is a deceiver?

D.2 *Is quantum mechanics really strange?* Throughout this account (beginning with its title) I have emphasized that I find quantum mechanics to be strange. My question here: Is quantum mechanics intrinsically weird, or do I find it weird only because of the way I was brought up? For example, in the Middle Ages most people were brought up believing the earth to be flat. The round earth model must have seemed extraordinarily strange to them when it was first broached. (For example, it must have seemed paradoxical that you could travel always due east and yet eventually arrive back at your starting point.) Yet today even children find nothing unnatural about the round earth because they have heard about it from infancy.

Another example comes from chemistry. Joseph Black (1728–1799) discovered carbon dioxide and a number of basic chemical facts.

Soon after Black's death, one of his contemporaries wrote in astonishment that

> He had discovered that a cubic inch of marble consisted of about half its weight of pure lime, and as much air as would fill a vessel holding six wine gallons. ... What could be more singular than to find so subtle a substance as air existing in the form of a hard stone, and its presence accompanied by such a change in the properties of that stone? ... It is surely a dull mind that will not be animated by such a prospect.

Today, few people consider simple chemical reactions to be "singular".

So what's the truth? Is quantum mechanics quite natural, but we were brought up to think otherwise? Or are chemical reactions in fact remarkable, but we were raised in a prosaic era?

D.3 *Layers of explanation.* In section 2.4 (page 9) I argued that the idea of explanation implied explanation in terms of more fundamental ideas, and that the most fundamental ideas could only be described and not explained. It was once thought that these deepest, simplest, most fundamental ideas ought to be "self evident". The fundamental ideas presented in this book have been very far from self evident. Is this a defect in the ideas presented here or a defect in the supposition of self evidence? (From the point of view of biological evolution, does it make sense that our brains should be hardwired to appreciate atomic phenomena?)

D.4 *Learning about quantum mechanics.* Describe your experience of learning about quantum mechanics. What motivated you to read this book? What questions did you have when you started it? Were those motivations satisfied and those questions answered? Did you learn the material by steady accumulation, or were there certain moments ("flashes of insight") when you suddenly came to understand large chunks of material that had been roving unprocessed about your mind? Different people learn in different ways. Which teaching techniques (lecture, conversation, reading, problem solving, film viewing, running computer simulations, etc.) do you think would be most effective for you in learning quantum mechanics? Is this the same answer that you would give for learning about, say, literature? Has this book changed your idea of the concept of "understanding" in science? What is your impression of your current understanding of quantum mechanics? (For example are you confused, disgusted, fascinated, satiated, all of the above?) Which unanswered questions are most important to you? Do you see any way that you can satisfy your continued curiosity?

D.5 *Rephrasing quantum mechanics.* Rewrite a section or a chapter of this book in your own terms. Make it clearer, or more correct, or more interesting than what I wrote. Explain briefly why your version is superior to mine. (Please send the author a copy of your revision and your explanation.)

D.6 *Can all authors be trusted?* In his book *Beyond the Quantum* (Macmillan, New York, 1986) Michael Talbot writes that the Aspect experiment forces the conclusion that "either objective reality does not exist and it is meaningless for us to speak of things or objects as having any reality above and beyond the mind of an observer, *or* faster than light communication with the future and the past is possible". (By the first alternative, he means standard quantum mechanics.) Is either branch of this dichotomy correct, or even internally consistent?

D.7 *What does "fundamental" mean?* Michael Horne and Anton Zeilinger (two of the proposers of the Greenberger–Horne–Zeilinger experiment) write that

> the greatest problem ... is to understand "why quantum mechanics?" Shouldn't a theory as fundamentally important as quantum mechanics follow from something deeper? We suggest that the fundamental elements of quantum mechanics may follow from a careful analysis of what it means to observe, to collect data, and to order them in such a way that physical laws can be constructed.

In section 2.4 of this book (page 9) I took exactly the opposite position, arguing that, by definition, a fundamental theory is one for which such questions cannot be answered. Which position, if either, do you support? Justify your preference.

D.8 *New, bizarre, or both?* In 1877, chemists were just beginning to learn how the arrangement of atoms within molecules could be deduced from chemical information. The distinguished chemist Hermann Kolbe called such attempts "hallucinations ... not many degrees removed from a belief in witches and from spirit-rapping". In 1980, distinguished physicist E.T. Jaynes referred to standard quantum mechanical ideas (such as those presented in this book) as "a violent irrationality ... more the character of medieval necromancy than science". What are your own reactions to quantum mechanics at this stage? Do you believe that Jaynes's reaction is more a rejection of the new and different or a rejection of the irrational? What of your own reaction?

D.9 *Quantum mechanics and Eastern mysticism.* In the 1970s two books appeared concerning the relation between quantum mechanics and mystical aspects of Eastern religion. These were Fritjof Capra's *The Tao of Physics* (Bantam, New York, 1975) written by a physicist, and Gary Zukav's *The Dancing Wu Li Masters* (Bantam, New York, 1979) written by a journalist. Read the two books and compare their treatments of both physics and religion. Can you find any errors in either book? To what extent can the differences in outlook and content of the two books be attributed to the professions of the two authors?

D.10 *Effect of quantum mechanics on culture.* What effect has the discovery of quantum mechanics had on broader human culture, such as philosophy, literature, politics, or popular thought? Are these effects due mostly to quantum mechanics or to misconceptions concerning quantum mechanics?

D.11 *Etymology.* How did the subject of this book come to be called "quantum mechanics"? After all, the word mechanics is usually associated with other activities. (Cartoon below courtesy of Sidney Harris.)

"ACTUALLY I STARTED OUT IN QUANTUM MECHANICS, BUT SOMEWHERE ALONG THE WAY I TOOK A WRONG TURN."

Appendix E
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R.P. Feynman, *The Character of Physical Law* (MIT Press, Cambridge, Massachusetts, 1965). Chapter 6, "Probability and uncertainty — the quantum mechanical view of nature", is the best one-hour summary of quantum mechanics that I know. It is the transcript of a lecture that was also recorded on film, and viewing the film is even better than reading the transcript. The video recording is distributed by Education Development Center, Inc.; 55 Chapel Street; Newton, Massachusetts 02158–1060.

P.C.W. Davies and J.R. Brown, *The Ghost in the Atom* (Cambridge University Press, Cambridge, UK, 1986). Interviews with quantum physicists at the popular level.

George Greenstein and Arthur G. Zajonc, *The Quantum Challenge: Modern Research on the Foundations of Quantum Mechanics* (Jones and Bartlett, Sudbury, Massachusetts, 1997). At the level of a junior or senior physics course, this book provides a superb account of experiments, but is sometimes murky in discussing the conceptual consequences of those experiments. For example, the authors say "the [Heisenberg] uncertainty principle prevents us from observing the trajectory of an electron ... in an atom", when they should say "an electron in an atom doesn't have a trajectory, so of course we can't observe it".

Jim Baggott, *The Meaning of Quantum Theory* (Oxford University Press, Oxford, UK, 1992). Another clear presentation at the mathematical level of a junior or senior physics course.

Leslie E. Ballentine, editor, *Foundations of Quantum Mechanics Since the Bell Inequalities* (American Association of Physics Teachers, College Park, Maryland, 1988). Reprints of articles, including an excellent annotated bibliography.

Hans Christian von Baeyer, *Taming the Atom: The Emergence of the Visible Microworld* (Random House, New York, 1992). A lyrical popular account. The word "taming" in the title carries the double meaning of "rendering useful" and "rendering familiar and visualizable".

S. Kamefuchi, editor, *Foundations of Quantum Mechanics in the Light of New Technology* (Physical Society of Japan, Tokyo, 1984); Daniel M. Greenberger and Anton Zeilinger, editors, *Fundamental Problems in Quantum Theory: A Conference Held in Honor of Professor John A. Wheeler* (The New York Academy of Sciences (Annals, volume 755), New York, 1995). Proceedings of two conferences devoted to the foundations of quantum mechanics. Much of these two volumes will be incomprehensible to the non-physicist, but they will show you that seasoned professionals as well as neophyte amateurs are fascinated and confused by the issues raised in this book.

Popularizations

Below are six conventional popularizations of quantum mechanics. I feel that each one of them is deficient either through oversimplification or through an emphasis on people rather than on nature. However, the authors of these books probably feel the same way about mine.

J.M. Jauch, *Are Quanta Real?* (Indiana University Press, Bloomington, Indiana, 1973). In many ways the best of the popularizations, but written before the full significance of the Einstein–Podolsky–Rosen paradox was understood.

Heinz Pagels, *The Cosmic Code: Quantum Physics as the Language of Nature* (Simon and Schuster, New York, 1982). An attempt to cover quantum mechanics, special and general relativity, statistical mechanics, elementary particle physics, and the history of each of these fields, all in one volume.

John R. Gribbin, *In Search of Schrödinger's Cat: Quantum Physics and Reality* (Bantam, New York, 1984). Describes both quantum mechanics and its history. Contains a few errors (pages 8, 167, 171, 229, 261, 265, and two on page 176). The author occasionally uses bizarre and misleading terminology, such as "the electron is not real" when he means "the electron does not have a definite position".

John R. Gribbin, *Schrödinger's Kittens and the Search for Reality: Solving the Quantum Mysteries* (Little, Brown and Company, Boston, 1995). Some history, some experimental tests, some alternative interpretations. Occasionally oversimplified to the point of error, as when, on pages 92–98, "probability" is used to mean "amplitude".

J.C. Polkinghorne, *The Quantum World* (Princeton University Press, Princeton, New Jersey, 1984). Written by a physicist turned priest. Nice description of the mathematical tools used by physicists to squeeze results out of the quantum theory. To my mind, this is more a book about how humans study nature than a book about nature, but if you want to find out what eigenvalues are and why you should care about them, then this is the book for you.

Alastair Rae, *Quantum Physics: Illusion or Reality?* (Cambridge University Press, Cambridge, UK, 1986). Contains six errors on the first three pages, but then improves.

Effect of quantum mechanics on culture

Caution: Some of these authors get the physics wrong. Some of them can't even distinguish between quantum mechanics and the theory of relativity!

Tom Stoppard, *Hapgood* (Faber and Faber, London, 1988). A sophisticated spy play involving quantum mechanics.

John Barth, *On With the Story* (Little, Brown and Company, Boston, 1996). In "Love Explained", one of the short stories in this collection, a character maintains that "More than Freudian psychology, more than Marxist ideology, quantum mechanics has been the Great Attractor of the second half of this dying century — even though, speaking generally, almost none of us knows beans about it."

"Wavefunctions for String Trio (four vignettes about the new [quantum] physics)" by John Tartaglia. Recording by Ensemble Capriccio published in 1999. The vignettes include "Bell's theorem" and "Schrödinger's cat".

Jane Hamilton, "When I began to understand quantum mechanics", *Harper's*, **279** (August 1989) 41–49. A short story involving quantum physics and a beauty pageant.

June Jordan, "Poem on the quantum mechanics of breakfast with Haruko", *The Nation*, **257** (5 July 1993) 40. A love poem.

Eric Kraft, *Where Do You Stop?* (Crown, New York, 1992). Cataloged by the Library of Congress under "Quantum theory — Fiction".

Robert Anton Wilson, *Schrödinger's Cat* (Simon and Schuster, New York, 1979). A novel.

Susan Strehle, *Fiction in the Quantum Universe* (University of North Carolina Press, Chapel Hill, North Carolina, 1992).

Robert Nadeau, *Readings for the New Book on Nature: Physics and Metaphysics in the Modern Novel* (University of Massachusetts Press, Amherst, Massachusetts, 1981).

Aage Peterson, *Quantum Physics and the Philosophical Tradition* (MIT Press, Cambridge, Massachusetts, 1968).

Lawrence Sklar, *Philosophy of Physics* (Westview Press, Boulder, Colorado, 1992).

Jonathan Powers, *Philosophy and the New Physics* (Meuthen and Company, London, 1982).

James T. Cushing and Ernan McMullin, editors, *Philosophical Consequences of Quantum Mechanics: Reflections on Bell's Theorem* (University of Notre Dame Press, Notre Dame, Indiana, 1989).

Appendix F

Skeleton Answers for Selected Problems

Be sure to read page 10 about the philosophy behind active learning and problem solving before using these skeleton answers.

2.1. Large force directed downward, small force directed downward.
2.2. A > B = D > C.
2.3. 2800 miles.
2.4. –0.38 inches.
2.5. Infinite number, all perpendicular to the arrow.

3.1. All of the atoms would leave at one deflection corresponding to a large positive projection.

4.1. (a): 3 inches, (b): –3 inches, (c), (d), and (e): 0 inches, (f): $3/\sqrt{2} = 2.121$ inches.
4.2. They would all leave the – exit.
4.3. Because of the qualifier "in general", the claim is consistent with situations in which the probability of one outcome is 1 and the probability of all the other outcomes is 0.
4.4. (2).
4.5. All –. Half + and half –.
4.6. Not at all.
4.7. No.
4.8. 3/4.
4.9. 1/2.
4.10. 3/4, 1/4.
4.13. In both cases, "It just *is* correct. I can tell you about experiments which show *that* it is correct, but I can't say *why* it is correct."

5.1. 1/72.
5.2. 1/10.
5.3. 5/6.
5.4. (a), (b), and (c): $1/2^{10}$, (d): $10/2^{10}$.
5.5. (a), (b), and (c): 1/2.
5.6. (c): 5/8, 6/8.
5.7. Hint: "Thirty days hath September ...".
5.9. (a): $1/(25 \times 10^{12})$, (b): $1/(5 \times 10^6)$, (c): $51/(5 \times 10^6)$, (d): $(52 \times 51/2)/(5 \times 10^6)$.

6.3. 1/4.
6.4. 7/9.

8.1. Length 12.07 inches, direction 1:30 or "northeast".
8.2. Either "all of them" or "none of them" are acceptable answers.

9.1. These phenomena happen even when only one atom is present in the apparatus.
9.2. (a): 1/4, (b): 1/4, (c): 1.
9.3. 50%, 50%, 0%, 100%, 50%, 0%, 50%, 0%, 12.5%.
9.6. (1) Measurement means someone looks. (2) An electron is a marble with a definite position, that goes through one hole or the other but neither you nor nature knows which.

10.2. If an atom's position were always definite, then quantal interference (experiment 9.3) would be much worse than a puzzle, it would be a logical contradiction. We are able to regain logical consistency only by abandoning the mental picture of an atom as a small, hard marble.

11.2. $1/\sqrt{2}$, $1/\sqrt{2}$, 0.
11.4. 1/4.

14.1. Yes, yes, no, no, no.

15.1. "Between release and detection, the electron is not at *any* point, because it doesn't have a position. Instead, it has amplitude to be at each of many points."

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