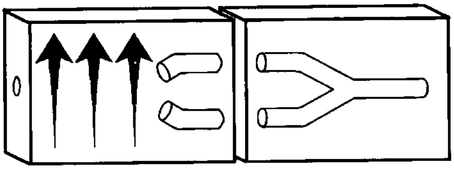
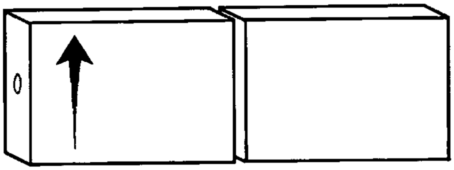
9

Quantal Interference

We have seen that quantum mechanics can only find probabilities and not certainties. Now we must find out how to work with these probabilities.* We will do this by examining the results of several experiments performed with a new instrument, the *interferometer* (also called an *analyzer loop*).

The interferometer is a Stern–Gerlach analyzer followed by plumbing that recombines the paths of atoms leaving from either exit. The design above is represented by the simple figure below. An interferometer must be constructed in such a way that the two branches are absolutely identical, whence it is impossible to tell by examining the outgoing atom which of the two branches it went through. For example, the two branches must have exactly the same length, because otherwise it would take an atom more time to traverse the longer branch. Because of this precise construction,





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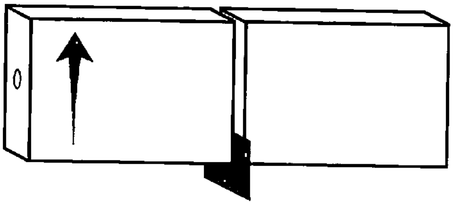
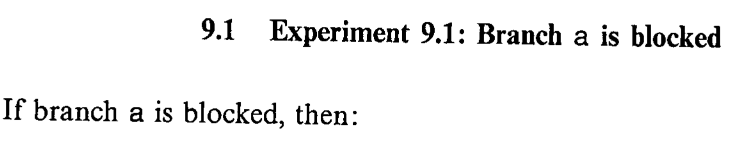
* This book presents the standard description of quantum mechanics. Other descriptions — notably that of David Bohm — are also possible. But, as required by the Einstein–Podolsky–Rosen effect, all of the viable alternative descriptions are either probabilistic or non-local or both.

when an atom leaves the interferometer it is in exactly the same state as it was when it entered. This holds regardless of the interferometer's orientation.

Thus the interferometer is an instrument that does nothing at all! The outgoing atom is the same as the incoming atom. It is hard to see why anyone would want to build one. Of course it can be made to do something useful by blocking one of its two branches. For example, in the interferometer below the lower branch is blocked, so it behaves just like a vertical Stern–Gerlach analyzer with its bottom exit blocked: not all of the incoming atoms will go out, but each one that does has $m_z = +m_B$.



I will describe several experiments using the apparatus sketched below. In all cases the input atom has $m_z = +m_B$ (it has been gathered from the + exit of a vertical analyzer not shown in the figure). The atom passes through a horizontal interferometer, and then it is analyzed with a vertical analyzer. An atom leaving the − exit of the vertical analyzer is considered output, while an atom leaving the + exit is ignored.



9.1 Experiment 9.1: Branch a is blocked

If branch a is blocked, then:

The probability of passing from input to intermediate is $\frac{1}{2}$.
The intermediate atom has $m_x = -m_B$.
The probability of passing from intermediate to output is $\frac{1}{2}$.
The overall probability of passing from input to output is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

9.2 Experiment 9.2: Branch b is blocked

If branch b is blocked, then the experiment proceeds exactly the same as experiment 9.1, except that the intermediate atom has $m_x = +m_B$.

9.3 Experiment 9.3: Neither branch is blocked

Analysis A. (Using the laws for compound probability.)

The atom goes from input to output either through branch a or through branch b.

It goes through branch a with probability $\frac{1}{4}$, or through branch b with probability $\frac{1}{4}$, so the overall probability of passing from input to output is $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

Analysis B. (Using the fact that an interferometer passes atoms unchanged.)

The probability of passing from input to intermediate is 1.

The intermediate atom has $m_z = +m_B$.

Any such atom leaves the + exit of the vertical analyzer, so ...

The overall probability of passing from input to output is 0.

A monumental disagreement! Which analysis is correct? Experiment confirms the result of analysis B, but what could possibly be wrong with analysis A? Certainly $\frac{1}{4}+\frac{1}{4} = \frac{1}{2}$ is correct, certainly the rule for compound probability (which is embodied in the second sentence) is correct. The only possible error is in the first sentence: "The atom goes either through branch a or through branch b." This common-sense assertion must be wrong! Indeed, if the atom passed through branch a then at the intermediate stage it would have a definite value of $m_x = +m_B$, but we know that this intermediate atom has a definite value of m_z so it *can't* have a definite value of m_x. The interferometer, which seemed so useless just a moment ago, is in fact an extremely clever way of correlating the position of an atom with its m_x: if $m_x = +m_B$, then the position is in branch a; if $m_x = -m_B$, then the position is in branch b. Since the incoming atom lacks a definite value of m_x, it must lack a definite position as well. The English language was invented by people who did not understand quantum mechanics, so it doesn't have an accurate concise way to describe what is going on in this experiment. The best approximate phrase is "the atom goes through both branches".

This conclusion seems patently absurd. Actually it is correct, and it seems absurd only if one thinks of an atom as being like a marble, only infinitely smaller and infinitely harder. In fact an atom is no more a small hard marble than an atom's magnetic needle is a pointy stick. These classical ideas are simply wrong when applied to very small objects. But I don't expect you to take my word for it. Let's perform an experiment in which we actually look at the two branches to see whether the atom is going through branch a, branch b, or both branches.

9.4 Experiment 9.4: Watching for atoms

In this experiment neither branch is blocked, but we train a powerful lamp on each branch to see whether the atom passes through branch a or through branch b. Inject an atom into the apparatus — a moment later we see a glint of light at branch b: the atom is going through branch b. Another atom, a glint at b again. Then a glint at a, then b again, then at a, etc. Never do we see, say, two weak glints, one at a and the other at b. "Ah ha!" you say, "So much for your metaphysical nonsense, Mr. Styer. Our observations show that the atom is going either through branch a or through branch b, and never through 'both', whatever *that* may mean."

True. But now look at the probability of passing from input to output. For unwatched atoms (experiment 9.3), that probability is zero. For watched atoms (experiment 9.4), that probability is $\frac{1}{2}$. If an atom *is* watched, then it *does* go either through branch a or through branch b, analysis A *is* correct, and half the atoms *do* leave the output! In fact, when the glint is seen at branch a then the intermediate atom has $m_x = +m_B$, as can be confirmed by replacing the vertical Stern–Gerlach analyzer with a horizontal one: an atom that causes a glint at branch a will always leave through the + exit of a horizontal analyzer, while one that causes a glint at branch b will always leave through the − exit.

Clearly a "watched" (or "observed") atom behaves differently from an unwatched atom. Much silliness has been written concerning the subject of precisely what constitutes an observation. Suppose, for example, that we train the lamps on the interferometer but turn our backs and don't look for the glints. Have the atoms been watched or haven't they? What if the glints are watched by cats rather than by human beings? Such questions are most easily answered by considering a parallel experiment. Suppose we turn our backs on the glints but record them on a movie. Now suppose the movie is played back, to either a human or a feline audience, one hour after the experiment is finished. Certainly by this time it is too late to change the way atoms exit from the vertical analyzer! In fact the significant question is not whether someone actually *sees* which branch an atom takes, but whether it is, in principle, *possible* to determine which branch an atom takes, regardless of whether any human actually takes advantage of that possibility. (Sometimes the term "registered" is used instead of "observed" or "measured" to emphasize that no human involvement is required.) From this perspective, the blocks in experiments 9.1 and 9.2 are simply ways to determine which branch the atom took: if the atom emerges while branch a is blocked, then it must have taken branch b. (I warn you, however, that it is not always easy to decide whether or not an observation is "in principle possible", nor to uncover the exact moment at which an observation is made.)

Perhaps you think that the "problem" with experiment 9.4 is that the atoms are being disturbed by the intense light. An atom is a tiny thing, after all, and perhaps the blast of light is simply pushing it around uncontrollably. This thought inspires the next two experiments.

9.5 Experiment 9.5: Watching for atoms at branch a only

In this case the intense light is trained only on branch a, so it cannot possibly disturb an atom that passes through branch b. As an atom passes through the interferometer there is either a glint at a, which means that the atom has passed through branch a, or else there is no glint at all, which means that the atom has passed through branch b. Since it is possible to determine which branch the atom passed through, the results are exactly the same as those of experiment 9.4.

9.6 Experiment 9.6: Watching for atoms with dim light

Although the light is dimmer, the glints are exactly the same! (This is because each glint corresponds to exactly one photon.) When the light is dim, however, some atoms pass through the interferometer without producing a glint at all. Careful analysis of the experimental results shows that an atom which produces no glint behaves just as if it were in experiment 9.3 (unwatched atoms), while one which does produce a glint behaves just as if it were in experiment 9.4 (watched atoms).

9.7 Is measurement magical?

How can the behavior of an atom depend upon whether or not it is being watched? Can't watching happen without the atom being affected? No. The only way to observe/measure/watch a system is to influence/disturb/alter it in some way. Consider, for example, a ball tossed upward in a room with ceiling lamps. If the lamps are off, the ball will ascend to a certain height. If the lamps are on, then the light will press down on the ball and it will attain a somewhat lower height. This effect is negligible if the ball is a baseball[†] but important if the ball is an atom, because it is much easier to push an atom around than a baseball. (Notice that it is the presence of light, not of watchers, in the room that makes the difference. Once again, the important issue is whether the observation is possible in principle, not whether a person — or a cat — happens to take advantage of that possibility.)

This is not to say that all questions concerning quantal measurement — and concerning its sister subject, the classical limit of quantum mechanics — are completely solved and pat. They are not. Consider the question of the Stern–Gerlach analyzer *vs.* the Stern–Gerlach interferometer. In the first device, the atom emerges from one exit or the other but not both. In the second device, the atom goes through one branch or the other or both. But the front half of an interferometer is exactly the same as an analyzer! How does the atom "know" that in the interferometer the two branches will ultimately be recombined?[‡] Questions like these are far more subtle than they appear, and are the subject of current investigation. Although measurement is not magical, it still holds mysteries.

9.8 Understanding

Whenever I lecture concerning the topic of this chapter, students approach me afterwards and say "I followed the lecture, but I just don't understand it." When I delve into exactly what is disturbing these students, it usually turns out to be one of two conceptual roadblocks: either the student simply finds that this behavior is unfamiliar and unexpected, or else (s)he is seeking a mechanism which underlies the behavior.[§]

This behavior certainly is unexpected, but that doesn't mean that it is wrong. If you were born in orbit in a space station and landed on earth for your sixteenth birthday, then you would find gravitational attraction unfamiliar and unexpected. But it is not wrong to feel that way. Indeed gravity truly is a mysterious force! Many people feel more comfortable with a new phenomenon if it is given a name. The strange attraction of remote bodies is called "gravity". Perhaps it will comfort you to know that the strange phenomenon described in this chapter is called "quantal interference".

[†] Indeed, the effect is small enough that many people don't know it exists. However, all science fiction buffs have read stories about spaceships driven by the sunlight reflected from huge gossamer sails.

[‡] This is the content of the so-called "Schrödinger's cat" paradox.

[§] Another discussion of the meaning of "understanding" in science is given by R.P. Feynman in *QED: The Strange Theory of Light and Matter* (Princeton University Press, Princeton, New Jersey, 1985), pages 9–10.

What is the mechanism that underlies quantal interference? People ask this question thinking that there is some explanation of the sort: "An atom is made up of two bricks held together with a rubber band, and when the rubber band hits the wall of branch a then the two bricks oscillate back and forth and ...". But an atom is not made up of bricks and rubber bands. Instead bricks and rubber bands are made up of atoms! The Einstein–Podolsky–Rosen arguments show that no local deterministic mechanism, no matter how intricate, can lead to the results of quantum mechanics. As far as anyone knows, there is no mechanism. This is simply the way the universe works.

9.9 References

The idea that interference lies at the heart of quantum mechanics was recognized from the the founding of the subject in the 1920s, but it has been emphasized most notably in theoretical treatments by Feynman. See, for example,

> R.P. Feynman, *The Character of Physical Law* (MIT Press, Cambridge, Massachusetts, 1965) chapter 6,
> R.P. Feynman, *QED: The Strange Theory of Light and Matter* (Princeton University Press, Princeton, New Jersey, 1985) pages 77–82,
> R.P. Feynman, R.B. Leighton, and M. Sands, *The Feynman Lectures on Physics* volume III: *Quantum Mechanics* (Addison-Wesley, Reading, Massachusetts, 1965) chapters 1 and 5,
> R.P. Feynman and A.R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965) chapter 1.

Interference experiments using photons have been performed in the laboratory for centuries. But laboratory (as opposed to "thought experiment") interferometers that use matter rather than light are relatively young. An accessible description of an early experiment is

> D.M. Greenberger and A.W. Overhauser, "The role of gravity in quantum theory", *Scientific American*, **242** (5) (May 1980) 66–76, 186.

This interferometer employed neutrons and its builders used it to investigate the effects of gravity. Interferometers using matter have grown steadily more sophisticated. This growth is reviewed in

> Barbara Levy, "Atoms are the new wave in interferometers", *Physics Today*, **44** (7) (July 1991) 17–20,

and it has culminated, at least for the moment, in the actual execution of the experiments suggested so long ago as theoretical exercises by Feynman:

A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, and H. Ezawa, "Demonstration of single-electron buildup of an interference pattern", *American Journal of Physics*, **57** (1989) 117–120,

R. Gähler and A. Zeilinger, "Wave-optical experiments with very cold neutrons", *American Journal of Physics*, **59** (1991) 316–324,

Michael S. Chapman, David E. Pritchard, *et al.*, "Photon scattering from atoms in an atom interferometer: Coherence lost and regained", *Physical Review Letters*, **75** (1995) 3783–3787,

E. Buks, R. Schuster, M. Heiblum, D. Mahalu, and V. Umansky, "Dephasing in electron interference by a 'which-path' detector", *Nature*, **391** (1998) 871–874.

The research questions concerning measurement and the classical limit, touched upon in section 9.7, are discussed in more detail and at various technical levels in

J.A. Wheeler and W.H. Zurek, editors, *Quantum Theory and Measurement* (Princeton University Press, Princeton, New Jersey, 1983) especially pages 184–185,

A.J. Leggett, "Schrödinger's cat and her laboratory cousins", *Contemporary Physics*, **25** (1984) 583–598,

Eric J. Heller and Steven Tomsovic, "Postmodern quantum mechanics", *Physics Today*, **46** (7) (July 1993) 38–46,

V.B. Braginsky and F.Ya. Khalili, "Quantum nondemolition measurements: the route from toys to tools", *Reviews of Modern Physics*, **68** (1996) 1–11,

Paul Kwiat, Harold Weinfurter, and Anton Zeilinger, "Quantum seeing in the dark", *Scientific American*, **275** (5) (November 1996) 72–78,

Serge Haroche, "Entanglement, decoherence and the quantum/classical boundary", *Physics Today*, **51** (7) (July 1998) 36–42.

Anyone who remembers the American presidential election of 1992 (Bush *vs.* Clinton *vs.* Perot) will enjoy the many insights, concerning both physics and politics, to be found in

N.D. Mermin, "Two lectures on the wave-particle duality", *Physics Today*, **46** (1) (January 1993) 9–11.

9.10 Sample problem

In the apparatus sketched on the next page, atoms with $m_z = +m_B$ are passed through a horizontal interferometer (number 1) then a vertical interferometer (number 2). If all branches are open, 100% of the incoming

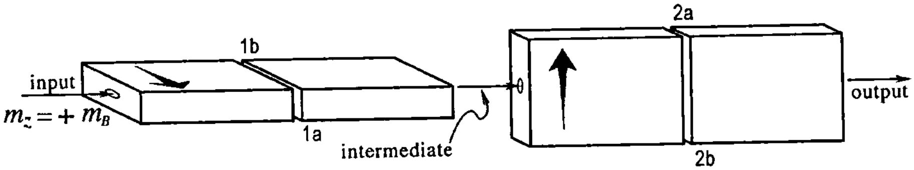


Fig. 9.1. Two interferometers. (Sample problem on page 71.)

atoms exit from the output. What percentage of the incoming atoms leave from the output if the following branches are blocked? (The atoms are not observed as they pass through the interferometers.)

(a)	2a	(d)	1b
(b)	2b	(e)	1b and 2a
(c)	1a	(f)	1a and 2b

Solution

Only two principles are needed to solve this problem: First, an atom leaving an unblocked interferometer leaves in the same state that it was in when it entered. Second, an atom leaving an interferometer that has one branch blocked leaves in the state specified by the branch through which it passed, regardless of what its entry state was. Use of these principles gives the solution on page 73. Notice that in changing from situation (a) to situation (e), you add blockage, yet you increase the output!

9.11 Problems

9.1 *Terminology.* Why are the phenomena described in this chapter better called "atom interference" rather than "the interference of atoms"?

9.2 *A different interference setup.* If the apparatus sketched on page 65 were changed so that atoms leaving the – exit were ignored, and atoms leaving the + exit were considered output, then what would be the probability of an atom passing from input to output if (a) branch a were blocked, (b) branch b were blocked, or (c) neither branch were blocked.

9.3 *Three interferometers.* Atoms with $m_z = +m_B$ pass through a horizontal interferometer, then a vertical interferometer, then a horizontal interferometer, as shown on page 74. What percentage of the incoming atoms leave from the output if the following branches are blocked? (The atoms are not observed as they pass through the interferometers.)

branches blocked	input state	branch taken through # 1	intermediate state	branch taken through # 2	output state	probability of input → output
none	$m_z = +m_B$	both	$m_z = +m_B$	a	$m_z = +m_B$	100%
2a	$m_z = +m_B$	both	$m_z = +m_B$	100% stopped at a	none	0%
2b	$m_z = +m_B$	both	$m_z = +m_B$	a	$m_z = +m_B$	100%
1a	$m_z = +m_B$	50% stopped at a 50% pass through b	$m_x = -m_B$	both	$m_x = -m_B$	50%
1b	$m_z = +m_B$	50% pass through a 50% stopped at b	$m_x = +m_B$	both	$m_x = +m_B$	50%
1b and 2a	$m_z = +m_B$	50% pass through a 50% stopped at b	$m_x = +m_B$	25% stopped at a 25% pass through b	$m_z = -m_B$	25%
1a and 2b	$m_z = +m_B$	50% stopped at a 50% pass through b	$m_x = -m_B$	25% pass through a 25% stopped at b	$m_z = +m_B$	25%

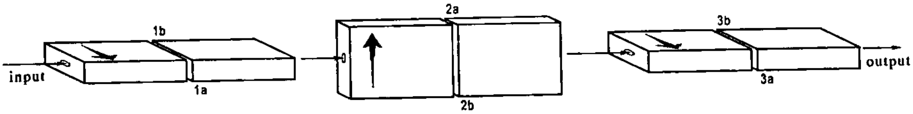


Fig. 9.2. Three interferometers. (Problem 9.3.)

(a) 3a
(b) 3b
(c) 2a
(d) 2b
(e) 1b
(f) 2a and 3b
(g) 1b and 3b
(h) 1b and 3a
(i) 1b and 3a and 2a

(Note that in going from situation (h) to situation (i) you get *more* output from *increased* blockage.)

9.4 *Paradox?*

(a) The year is 1492, and you are discussing with a friend the radical idea that the earth is round. "This idea can't be correct," objects your friend, "because it contains a paradox. If it were true, then a traveler moving always due east would eventually arrive back at his starting point. Anyone can see that that's not possible!" Convince your friend that this paradox is not an internal inconsistency in the round-earth idea, but an inconsistency between the round-earth idea and the picture of the earth as a plane, a picture which your friend has internalized so thoroughly that he can't recognize it as an approximation rather than the absolute truth.

(b) The year is 1992, and you are discussing with a friend the radical idea of quantal interference. "This idea can't be correct," objects your friend, "because it contains a paradox. If it were true, then an atom passing through branch a would have to know whether branch b were open or blocked. Anyone can see that that's not possible!" Convince your friend that this paradox is not an internal inconsistency in quantum mechanics, but an inconsistency between quantal ideas and the picture of an atom as a hard little marble that always has a definite position, a picture which your friend has internalized so thoroughly that he can't recognize it as an approximation rather than the absolute truth.

(If you cannot solve this problem now, then come back to it after reading section 15.2, "What does an electron look like?")

9.5 *Definite position.* "It is absurd," Mr. Parker says, "to think that an atom might not have a definite position. It's not just atoms and positions, but *anything* must have a definite value for *any* of its attributes." You know that a glass prism splits white light up into its component colors. Convince Mr. Parker that a prism doesn't have a definite color.

9.6 *Misconceptions.* In his book *In Search of Schrödinger's Cat*, John Gribbin describes an experiment similar to our interferometer experiments, and concludes that "unless someone looks, nature herself does not know which hole the electron is going through". Which two misconceptions are embodied in this sentence?