

White dwarf stars

a. $N = \frac{M}{m_e + m_p} \approx \frac{M}{m_p}$.

b. The exact gravitational self energy is

$$\frac{1}{2} \int d^3r_1 \int d^3r_2 \left(-\frac{G\rho(\vec{r}_1)\rho(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|} \right).$$

By dimensional analysis, an estimate for the gravitational self energy of a sphere of radius R , mass M , and unknown mass distribution is

$$-c \frac{GM^2}{R}.$$

c.

$$\text{KE of ground state} = \frac{3}{5} N \mathcal{E}_F = \frac{3}{5} N \frac{\hbar^2 k_F^2}{2m_e}$$

$$k_F = \left(3\pi^2 \frac{N}{V} \right)^{1/3} = \left(3\pi^2 \frac{N}{(4/3)\pi R^3} \right)^{1/3} = \frac{1}{R} \left(\frac{9\pi}{4} N \right)^{1/3}$$

$$\begin{aligned} \text{KE} &= \frac{3}{10} \left(\frac{9\pi}{4} \right)^{2/3} \frac{\hbar^2 N^{5/3}}{m_e R^2} \quad [\dots \text{use } N = M/m_p \dots] \\ &= \frac{9}{20} \left(\frac{3\pi^2}{2} \right)^{1/3} \frac{\hbar^2 M^{5/3}}{m_e m_p^{5/3} R^2} \end{aligned}$$

d.

$$c \frac{GM^2}{2R} = \frac{9}{20} \left(\frac{3\pi^2}{2} \right)^{1/3} \frac{\hbar^2 M^{5/3}}{m_e m_p^{5/3} R^2}$$

$$\begin{aligned} MR^{1/3} &= \frac{2}{cG} \left(\frac{9}{20} \right) \left(\frac{3\pi^2}{2} \right)^{1/3} \frac{\hbar^2}{m_e m_p^{5/3}} \quad [\dots \text{use } c = 3/5 \dots] \\ &= \frac{3}{2} \left(\frac{3\pi^2}{2} \right)^{1/3} \frac{\hbar^2}{m_e m_p^{5/3}} \\ &= 2.87 \times 10^{17} \text{ kg}^{1/3} \text{m} \end{aligned}$$

e. 22,700 km (compare the sun's radius of 696,000 km)

f. For neutron stars, the derivation is identical except that m_e and m_p are both replaced by the mass of a neutron, which is nearly equal to m_p . Thus for a neutron star,

$$MR^{1/3} = (2.87 \times 10^{17} \text{ kg}^{1/3} \text{m}) \left(\frac{m_p}{m_e} \right).$$

Applying this relationship to a neutron star with the mass of the sun gives a radius of 12.4 km.