Ideal paramagnet, take three

From the problem "Entropy of a spin system" . . .

$$S(E,H,N) = -k_B N \left\{ \left[\tfrac{1}{2}(1+u)\right] \ln\left[\tfrac{1}{2}(1+u)\right] + \left[\tfrac{1}{2}(1-u)\right] \ln\left[\tfrac{1}{2}(1-u)\right] \right\} \tag{1}$$

where

$$u = \frac{E}{NmH}.$$

And from the definitions

$$-\frac{\mu}{T} = \frac{\partial S}{\partial N} \quad \text{and} \quad \frac{1}{T} = \frac{\partial S}{\partial E}$$

we have

$$\mu(E,H,N) = -\frac{\partial S}{\partial N} \bigg/ \frac{\partial S}{\partial E}.$$

We call the stuff in curly brackets in equation (1) by the name $\{\sim\}$. Then

$$\begin{aligned}
\frac{\partial\{\sim\}}{\partial u} &= \tfrac{1}{2}\ln\left[\tfrac{1}{2}(1+u)\right] + \tfrac{1}{2} - \tfrac{1}{2}\ln\left[\tfrac{1}{2}(1-u)\right] - \tfrac{1}{2} \\
&= \tfrac{1}{2}\ln\left[\frac{1+u}{1-u}\right], \\
\frac{\partial S}{\partial N} &= -k_B\{\sim\} - k_B N \frac{\partial\{\sim\}}{\partial u}\frac{\partial u}{\partial N} \\
&= -k_B\{\sim\} - k_B N \tfrac{1}{2}\ln\left[\frac{1+u}{1-u}\right]\left(-\frac{E}{N^2 mH}\right) \\
&= -k_B\{\sim\} + k_B \frac{u}{2}\ln\left[\frac{1+u}{1-u}\right], \\
\frac{\partial S}{\partial E} &= -k_B N \frac{\partial\{\sim\}}{\partial u}\frac{\partial u}{\partial E} \\
&= -k_B N \tfrac{1}{2}\ln\left[\frac{1+u}{1-u}\right]\left(\frac{1}{NmH}\right) \\
&= -k_B \frac{1}{2mH}\ln\left[\frac{1+u}{1-u}\right].
\end{aligned}$$

Thus

$$\begin{aligned}
\mu = -\frac{\partial S}{\partial N} \bigg/ \frac{\partial S}{\partial E} &= \frac{-k_B\{\sim\} + k_B\frac{u}{2}\ln\left[\frac{1+u}{1-u}\right]}{k_B\frac{1}{2mH}\ln\left[\frac{1+u}{1-u}\right]} \\
&= 2mH\left[\frac{-\{\sim\} + \frac{u}{2}\ln[\quad]}{\ln[\quad]}\right] \\
&= mH\left[\frac{-2\{\sim\} + u\ln[\quad]}{\ln[\quad]}\right].
\end{aligned}$$

Meanwhile

$$\begin{aligned}
\{\sim\} &= \left[\tfrac{1}{2}(1+u)\right]\ln\left[\tfrac{1}{2}(1+u)\right] + \left[\tfrac{1}{2}(1-u)\right]\ln\left[\tfrac{1}{2}(1-u)\right] \\
&= \left[\tfrac{1}{2}(1+u)\right]\ln\tfrac{1}{2} + \left[\tfrac{1}{2}(1+u)\right]\ln(1+u) + \left[\tfrac{1}{2}(1-u)\right]\ln\tfrac{1}{2} + \left[\tfrac{1}{2}(1-u)\right]\ln(1-u) \\
&= \ln\tfrac{1}{2} + \left[\tfrac{1}{2}(1+u)\right]\ln(1+u) + \left[\tfrac{1}{2}(1-u)\right]\ln(1-u).
\end{aligned}$$

So

$$\begin{aligned}\mu &= mH\left[\frac{2\ln 2-(1+u)\ln(1+u)-(1-u)\ln(1-u)+u\ln(1+u)-u\ln(1-u)}{\ln[\quad]}\right] \\ &= mH\left[\frac{2\ln 2-\ln(1+u)-\ln(1-u)}{\ln(1+u)-\ln(1-u)}\right].\end{aligned}$$

Here's a graph: