Magnetic systems

\[
E(S, H) \\
\frac{dE}{dT} = T \frac{dS}{dT} - M \frac{dH}{dT}
\]

\[\downarrow \text{Legendre transformation}\]

\[
F(T, H) = E - TS \\
dF = -S dT - M dH \\
\text{Maxwell relation} \quad \frac{\partial S}{\partial H}_T = \frac{\partial M}{\partial T}_H
\]

\[\downarrow \text{Legendre transformation}\]

\[
A(T, M) = F + MH \\
dA = -S dT + H dM \\
\text{Maxwell relation} \quad - \frac{\partial S}{\partial M}_T = \frac{\partial H}{\partial T}_M
\]

Note particularly the physical/experimental meaning of the first Maxwell relation: To find the quantity on the left, you would measure the heat absorbed by the system as the applied magnetic field is changed while in a thermostatic bath: a very “heat-like” measurement. To find the quantity on the right, you would measure the magnetization of the sample using the same technique that you would use to measure the magnetization of a scout compass needle: a very “E\&M-like” measurement. It’s not immediately clear that the two quantities even have the same dimensions! Yet these two very different measurements are guaranteed to give always the same result.