

Accessible configurations of a spin system

a. Maximum possible energy is NmH — all spins down.

Minimum possible energy is $-NmH$ — all spins up.

Spacing between energy levels is $2mH$ — the energy to flip one spin.

b. The number of microstates (configurations) consistent with, say, five down spins is

$$\frac{N(N-1)(N-2)(N-3)(N-4)}{5!} = \frac{N!}{5!(N-5)!}.$$

In general, the number of microstates consistent with n_{\downarrow} down spins is the binomial coefficient

$$\frac{N!}{n_{\uparrow}!n_{\downarrow}!}.$$

These microstates correspond to the energy level with energy

$$E = -(n_{\uparrow} - n_{\downarrow})mH.$$

c. The number of configurations in this energy range is approximately the number of energy levels in the range times the number of configurations in a typical energy level. If n_{\uparrow} and n_{\downarrow} represent typical values for a level within this range, then

$$\Omega \approx \left(\frac{N!}{n_{\uparrow}!n_{\downarrow}!} \right) \left(\frac{\Delta E}{2mH} \right).$$

To write this as a function of macroscopic quantities only, we must solve for n_{\uparrow} and n_{\downarrow} in terms of E , N , and H . First write

$$\begin{aligned} n_{\uparrow} + n_{\downarrow} &= N \\ n_{\uparrow} - n_{\downarrow} &= -E/mH, \end{aligned}$$

and then solve to find

$$\begin{aligned} n_{\uparrow} &= \frac{1}{2}(N - E/mH) \\ n_{\downarrow} &= \frac{1}{2}(N + E/mH). \end{aligned}$$

Putting all this together gives

$$\Omega(E, \Delta E, H, N) \approx \frac{N!}{\left[\frac{1}{2}(N + E/mH)\right]! \left[\frac{1}{2}(N - E/mH)\right]!} \frac{\Delta E}{2mH}.$$