Remarks on Granville Sewell’s treatment of “Entropy and Evolution”

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Professor Granville Sewell’s essay1 “Entropy and evolution”, appears, on its surface, to disagree with my article2 of the same title. The thrust of Sewell’s essay is that biological evolution is not prohibited by the law of entropy increase as we know it today, but that we have not yet discovered all the scientific laws. There is no disagreement here: Sewell concurs with me on both of these points.3

But in reaching these points Sewell presents a number of misleading, irrelevant, or erroneous claims. This short piece lists only eight of the errors made by Professor Sewell, then concludes with a general comment.

1. In his very first line, Sewell invokes “the spectacular local decreases in entropy that occurred on Earth as a result of the origin and evolution of life and the development of human intelligence.” In fact, however, this decrease is far from spectacular: Reference 2 shows that it is less than 3.02 J/K each second, which is minute when compared to the entropy that the Earth gains from the Sun, and loses to the cosmic microwave background, of $4.20 \times 10^{14}$ J/K each second.

[A quantity can be “large” or “small”, “spectacular” or “commonplace”, only in comparison to another quantity with the same dimensions. In the summer of 2013, I spent 67 days hiking 1126 kilometers on the Idaho Centennial Trail. That distance is “spectacular” compared to the diameter of an atom, yet “minute” compared to the diameter of a star.]

2. At the top of the left column of page 2, Sewell says that “entropy” is most often used simply as a synonym for disorder.” Indeed, on 6 January 2015 the New York Times editorial board used the term “entropy” in exactly this manner when it wrote that “Mayor Bill de Blasio has been in office barely a year, and already forces of entropy are roaming the streets, turning their backs on the law, defying civil authority and trying to unravel the social fabric.” In casual writing “entropy” can mean “disorder”, and “disorder” can mean “chaos”. But “disorder” is emphatically not the meaning of the word “entropy” in science.4 The Times is clearly not using the scientific meaning of entropy, because it mentions “forces of entropy”: force is measured in units of joule/meter whereas entropy is measured in units of joule/kelvin, so, in the scientific meanings of these two words, there can be no such thing as “forces of entropy”.

In the scientific sense of the word “entropy”, there are experimental procedures for measuring entropy and formulas for calculating entropy. Libraries contain books5 that tabulate entropy values for various substances at various temperatures and various pressures. In contrast, there is no experimental procedure for measuring

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3. All scientists agree that we have not yet discovered all the scientific laws: It is the job of a scientist to discover more about nature. If everything about nature were already known, then there would be no more to discover, so no one would have the job of performing that discovery, so there would be no scientists. The very fact that scientists exist implies that our knowledge of science is incomplete.
4. See, for example, Frank L. Lambert, “Shuffled Cards, Messy Desks, and Disorderly Dorm Rooms — Examples of Entropy Increase? Nonsense!”, Journal of Chemical Education 76, 1385–1387 (October 1999), and Professor Lambert’s essays at http://entropysite.oxy.edu/.
5. For example, Ihsan Barin, Thermochemical Data of Pure Substances (VCH Publishers, Weinheim, Germany, 1995).
disorder; there are no formulas for calculating disorder; there are no books tabulating numerical values of disorder. There is not even general agreement on what constitutes disorder: Some observers consider the artwork of Jackson Pollock to be disorganized scribbling. Others find those same artworks to be pregnant with structure and meaning.

Smash some ice cubes and place the shards into a bowl on a kitchen table. They melt into smooth, uniform liquid water. The jumble of ice shards is disorderly, the homogeneous liquid water is orderly. Yet the ice shards have the lower entropy.\footnote{Thomas A. Moore and Daniel V. Schroeder, “A different approach to introducing statistical mechanics”, \textit{American Journal of Physics} \textbf{65}, 26–36 (1997).}

Most words have multiple meanings, and it is not uncommon for a word to have one meaning in science and another meaning in daily life: when a music critic writes that “the piano sonata was performed with enormous energy”, the critic doesn’t mean that the performance had a high value of

$$\sum_i \frac{1}{2}m_i v_i^2 + \sum_i m_i g h_i + \frac{1}{2} \sum_{i,j} U_{i,j}.$$ 

But it \textit{is} uncommon — and an error — for a scientist such as Professor Sewell to deliberately use an unscientific meaning of a scientific term in a scientific context.

3. In the middle of the left column of page 2, Sewell mentions a “rate of decrease in entropy associated with biological evolution [of] less than 302 Joules/degree Kelvin/second”.

a. What does $A/B/C$ mean? Does it mean $(A/B)/C = A/BC$ or does it mean $A/(B/C) = AC/B$?

b. The “degree Kelvin” went out of existence as a unit at the thirteenth General Conference on Weights and Measures\footnote{Resolution 3.} in October 1967. Since that date, the unit of temperature has been the “kelvin” (symbol K) rather than the “degree Kelvin” (symbol °K).

I admit that these are picky details, and that I can divine Professor Sewell’s meaning. But Sewell should not have to rely on divination: he ought to get his details straight. I would never let a first-year physics student get away with either of these errors. (Ask my students if you don’t believe me.)

4. In the right column of page 2, Sewell quotes Andy McIntosh that “all these authors [Styer and Bunn] are making the same assumption — viz. that all one needs is sufficient energy flow into a [non-isolated] system and this will be the means of increasing the probability of life developing in complexity and new machinery evolving.”

But the truth is that neither of the authors mentioned make the assumption that “all one needs is sufficient energy flow”. In fact, in the last paragraph of my paper (reference 2) I anticipate McIntosh’s objection and point out that it is an error: “The second law of thermodynamics \textit{permits} but does not \textit{require} evolution. For example, the second law of thermodynamics holds on the Moon, yet biological evolution doesn’t occur there.”
5. Also on page 2, Sewell claims that “the Boltzmann formula \( S = k_B \ln \Omega \) relates the thermal entropy of an ideal gas [macro]state \( S \) to the number of possible microstates \( \Omega \)”. The misconception that Boltzmann’s microcanonical formula for entropy applies only to the ideal gas is utterly mistaken. This formula has been successfully applied to paramagnets, to vibrating molecules, to the Einstein model for lattice vibrations. It can be shown equivalent (in the thermodynamic limit) to the formula for Helmholtz potential \( F = -k_B T \ln Z \) (where \( Z \) is the canonical partition function) which in turn has been applied to Ising magnets, to chemical reactions, to the Debye and other models for lattice vibrations, to phase transitions and phase coexistence, to non-ideal gases, to liquids, to crystals, yes, even to light in thermal equilibrium (black body radiation).

6. On the left column of page 3, Sewell considers the situation of “an isolated steel object, [in which] the chromium distribution is initially non-uniform, and becomes more uniform with time, until the chromium concentration is constant throughout. Then, the chromium distribution starts to become non-uniform again.” He claims that “everyone agrees that the second law is being obeyed during the first stage, and violated during the last stage.”

This claim is false.

A glance at the chromium-iron phase diagram\(^8\) demonstrates instantly, to all who can interpret binary alloy phase diagrams,\(^9\) that many chromium-iron mixtures are not uniform at equilibrium, but instead segregate into two components, one chromium-rich and the other iron-rich. For the case of, say, 30% chromium by mass at a temperature 600°C, uniform distribution

\(^8\)See, for example, http://www.calphad.com/iron-chromium.html.

\(^9\)See, for example, William D. Callister and David G. Rethwisch, *Materials Science and Engineering: An Introduction* chapter 9.
of chromium in iron is a non-equilibrium situation... if such a chromium-iron mixture started uniform, the materials would spontaneously diffuse into an inhomogeneous segregated situation with some of the alloy in form $\alpha$ containing about 22% chromium and some of it in form $\sigma$ containing about 46% chromium. For the four phase diagram regions marked “$\alpha + \sigma$” or “$\gamma + \alpha$” or “Liquid + $\alpha$”, Professor Sewell’s claim is exactly backwards.

Apparently Sewell has fallen into the misconception that equilibrium implies homogeneity. Yet everyone knows that a mixture of olive oil and vinegar forms an inhomogeneous equilibrium at room temperature: one segregated phase is vinegar-rich with a bit of dissolved oil; the other is oil-rich with a bit of dissolved vinegar. Anyone familiar with Italian salad dressing knows that equilibrium macrostates are not necessarily homogeneous.

On the basis of Sewell’s misconception\textsuperscript{10} that chromium must be distributed uniformly at equilibrium, he proposes the existence of “chromium entropy” as the first of many new entropy-like quantities. You will not find books like reference 5 tabulating values of “chromium entropy” or “sulfur entropy” or “helium entropy” for the simple reason that Sewell’s proposal is based on an error.

7. In pages 3 and 4, Sewell lists a number of unlikely events (for example, a tornado converting a pile of rubble into houses and cars), and then erroneously suggests that these improbable events ought to be prohibited by either the existing second law of thermodynamics or by some potential future generalization. One can readily add to his list of improbable events:

a. It is improbable that my checking account will ever hold $1$ million or more — but if it does, that would not violate the second law of thermodynamics.

b. It is improbable that Arnold Schwarzenegger will ever become President of the United States — and if he does, that would violate the U.S. Constitution’s provision that the President must be a “natural born Citizen”, but that would not violate the second law of thermodynamics.

c. It is improbable that Professor Sewell will acknowledge that he and I agree about entropy (as currently understood) and evolution — but if he does, that would not violate the second law of thermodynamics.

8. On page 5, Sewell suggests that the potential future generalization of the second law of thermodynamics might look something like: “Natural (unintelligent) forces do not do macroscopically describable things that are extremely improbable from the microscopic point of view.”

It is a common occurrence for snow to crystallize, for hurricanes to grow out of undifferentiated breezes, for geysers to spout, for waves to develop on a once-calm lake, for acorns to become oak trees, and for dust bunnies to coalesce under beds. All of these occurrences are natural, all are macroscopically describable, and all are extremely improbable from the microscopic point of view.

\textsuperscript{10}In my experience, the misconception that “equilibrium implies homogeneity” is related to the misconception (#5 above) that “thermodynamics applies only to the ideal gas”. And indeed, for a classical ideal gas in the absence of external forces equilibrium macrostates are homogeneous. But even for a classical ideal gas in the presence of an external force, equilibrium macrostates are in general inhomogeneous. Consider a container of ideal gas 100 kilometers high with its base at sea level: at equilibrium, the bottom of the container is dense but its top is near-vacuum.
General comment. Professor Sewell agrees with me that the second law of thermodynamics, as presently understood, does not prohibit biological evolution on Earth. But clearly something more is desired: The second law of thermodynamics does not prohibit biological evolution on the Moon, yet that hasn’t happened. The second law of thermodynamics does not prohibit a tornado from converting a pile of rubble into houses and cars, yet that hasn’t happened. The second law of thermodynamics is valuable in that it rules out some conceivable things, yet some of the things permitted by the second law also don’t happen. Is there some deeper law of nature that will catch all the things that happen and exclude all the things that don’t happen?

Perhaps there is. Consider an isolated system of $N$ point particles with masses $m_i$, positions $\vec{x}_i(t)$, and velocities

$$\vec{v}_i(t) = \frac{d\vec{x}_i}{dt}.$$  

Suppose the particle interactions are described through the time-independent potential energy function

$$U(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N).$$

For notational convenience, exclude velocity-dependent forces (such as magnetism), relativistic effects, and quantal effects.

It is well-known that not everything is possible for this system. For example, the energy

$$E = \sum_i \frac{1}{2}m_i \vec{v}_i(t) \cdot \vec{v}_i(t) + U(\vec{x}_1(t), \vec{x}_2(t), \ldots, \vec{x}_N(t))$$

can neither increase nor decrease. The total momentum

$$\vec{P} = \sum_i m_i \vec{v}_i(t)$$

can neither increase nor decrease. The total angular momentum

$$\vec{L} = \sum_i m_i \vec{x}_i(t) \times \vec{v}_i(t)$$

can neither increase nor decrease. These three results are remarkable: the right-hand sides involve functions dependent on time $t$, but these many time dependencies all cancel out so that the left-hand sides are independent of time!

However, these three remarkable facts don’t tell us everything we might want to know about the system. For example, they can be used to show that the Earth orbits the Sun, but they can’t show that the shape of that orbit is an ellipse. To find more about the system, we can introduce additional quantities. In 1870, Rudolf Clausius found that the quantity

$$G(t) = \sum_i m_i \vec{v}_i(t) \cdot \vec{x}_i(t),$$

11 Including, alas, the $1 million in my checking account.
12 See, for example, L.D. Landau and E.M. Lifshitz, Mechanics, section 15, “Kepler’s problem.”
while not constant in time, could nevertheless be exploited to reveal important information about the system. He named this quantity “the virial”. Two years later, Ludwig Boltzmann found that that quantity

\[ H(t) = \sum_i \ln(h_0^3/a_i(t)), \]

where \( h_0 \) is an arbitrary constant with the dimensions of action and \( a_i(t) \) is the volume of the Voronoi polyhedron surrounding particle \( i \) in phase space, was also valuable. He unimaginatively named this quantity “\( H \)”, and showed that \(-k_B H(t)\) is related to (although not equal to) the entropy.

I don’t for a minute believe that we discovered the last interesting combination of quantities in 1872. Professor Sewell and I both believe that further interesting combinations await discovery.\(^{13}\) Sewell speculates that some to-be-discovered combination\(^{14}\) might prohibit or restrict biological evolution, but provides absolutely no basis or rationale for his speculation. Most scientists agree with me that it’s useless to speculate: we will know only after they are discovered. But Sewell agrees with me that no theorem discovered to date prohibits biological evolution: This is precisely why he has to speculate; this is precisely why he must invoke the as-yet-undiscovered (and, as it happens, never-to-be-discovered) entity of “chromium entropy”.

But stepping beyond “interesting combinations”, is there anything that can tell us with certainty what behavior will happen and what behavior will not? Indeed there is. The \( N \) differential equations

\[ m_i \frac{d^2 \vec{x}_i(t)}{dt^2} = -\vec{\nabla}_i U(\vec{x}_1(t), \vec{x}_2(t), \ldots, \vec{x}_N(t)), \]

with their associated initial conditions, predict what will happen and prohibit all other possibilities. These so-called “equations of motion” were discovered by Isaac Newton in 1687.

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\(^{14}\)Sewell does not provide a formula or even a name for his to-be-discovered combination, but claims that it is similar to “specified complexity” — another combination that lacks a formula.