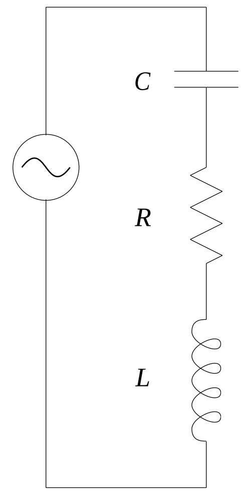
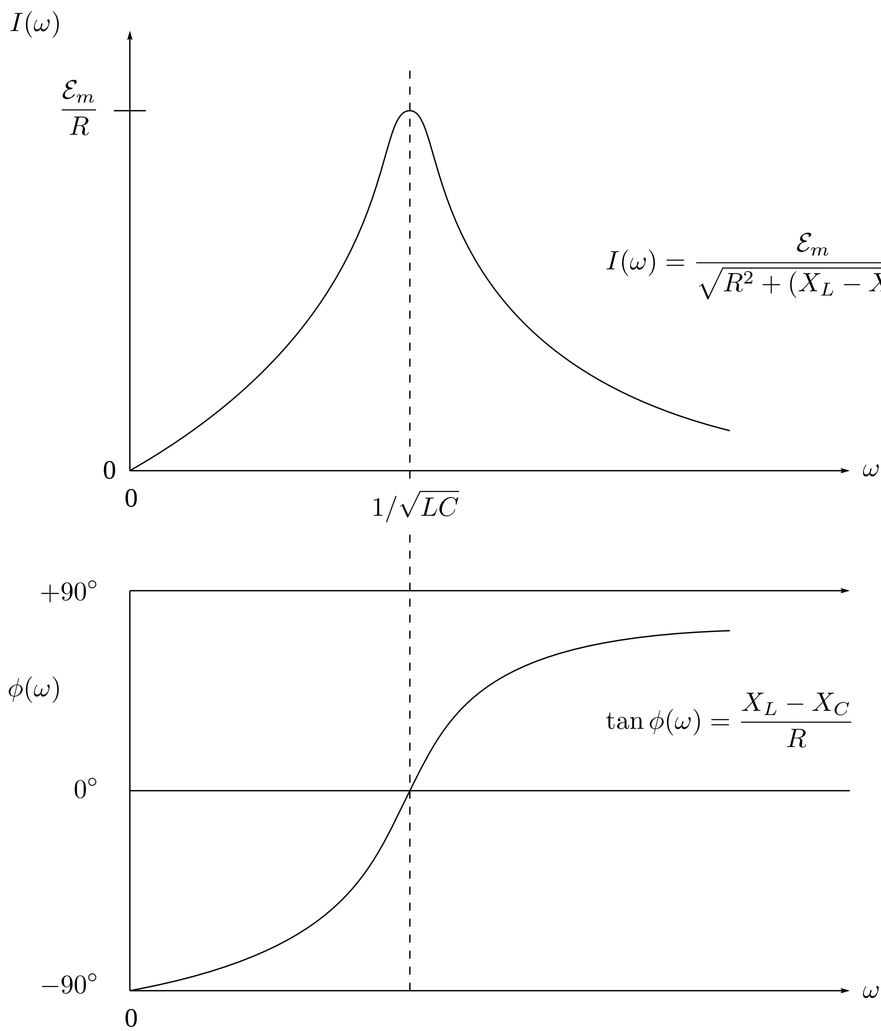
The LCR circuit

define reactance: $X_C = \dfrac{1}{\omega C}; \quad X_L = \omega L$

$$\mathcal{E}(t) = \mathcal{E}_m \sin(\omega t)$$



$$i(t) = I(\omega)\sin(\omega t - \phi(\omega))$$



$$I(\omega) = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\tan\phi(\omega) = \frac{X_L - X_C}{R}$$

Slow changes $\left(\text{low frequencies; } \omega \ll 1/\sqrt{LC}\right)$

- capacitative reactance dominates $\Rightarrow I(\omega)$ small
- $v_C(t) \approx \mathcal{E}(t)$
- $\dfrac{q(t)}{C} \approx \mathcal{E}_m \sin(\omega t)$
- $i(t) = \dfrac{dq(t)}{dt} \approx C\mathcal{E}_m \omega \cos(\omega t)$
- c.f. $i(t) = I \sin(\omega t - \phi) \Rightarrow \phi \approx -90^\circ$

Fast changes $\left(\text{high frequencies; } \omega \gg 1/\sqrt{LC}\right)$

- inductive reactance dominates $\Rightarrow I(\omega)$ small
- $v_L(t) \approx \mathcal{E}(t)$
- $L\dfrac{di(t)}{dt} \approx \mathcal{E}_m \sin(\omega t)$
- $i(t) = \displaystyle\int \dfrac{di(t)}{dt}\, dt \approx -\dfrac{\mathcal{E}_m}{\omega L} \cos(\omega t)$
- c.f. $i(t) = I \sin(\omega t - \phi) \Rightarrow \phi \approx +90^\circ$

At resonance $\left(\omega = 1/\sqrt{LC}\right)$

- $v_C(t)$ and $v_L(t)$ can both be quite large, but
- $v_C(t) = -v_L(t)$ so $v_R(t) = \mathcal{E}(t)$
- $\Rightarrow I(\omega) = \mathcal{E}_m/R$ (a maximum)
- $\phi(\omega) = 0^\circ$