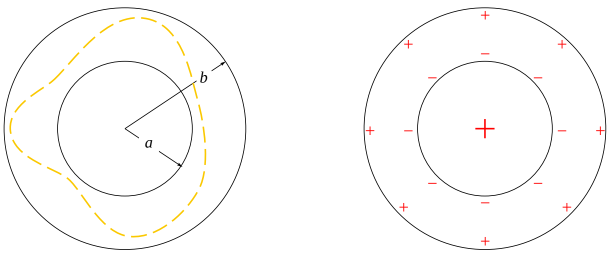
A conductor with a cavity enclosing a charge

In parts (a) through (d), the situation is spherically symmetric. Thus by standard symmetry arguments (i) the electric field $\vec{E}(\vec{r})$ must be radially directed and depend on the magnitude $|\vec{r}|$ only and likewise (ii) any surface charge density on the conductor must be uniformly distributed.

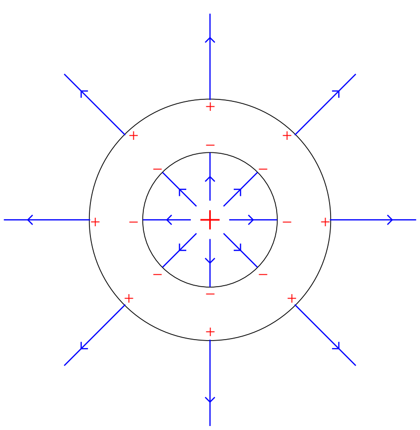
(a.)



Apply Gauss's law to the surface indicated by the dashed yellow line in the left figure above. Because $\vec{E} = 0$ inside the conductor, $\Phi = 0$ and hence $Q_{\text{inside}} = 0$. Thus the existence of a charge $+q$ within the cavity implies that there must also be negative charge $-q$ somewhere within the dashed yellow surface. The only place for it to be is on the inner surface of the conductor (uniformly distributed, because of symmetry). But the conducting shell carries no net charge, so the $-q$ on the inner surface must be compensated for by $+q$ elsewhere on the conductor. The only place for it be is on the outer surface (uniformly distributed, of course)!

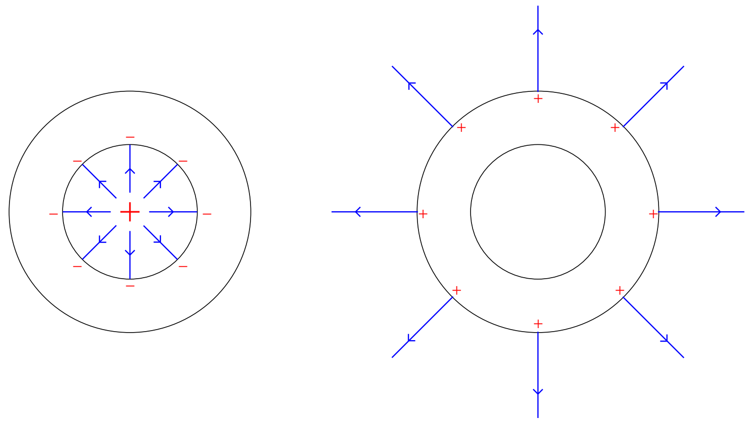
(b.) The charge distribution is a "spherically symmetric onion", so by superposition the magnitude of the electric field is

$$E(r) = \begin{cases} \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2} & r < a \\ 0 & a < r < b \\ \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2} & b < r \end{cases}$$



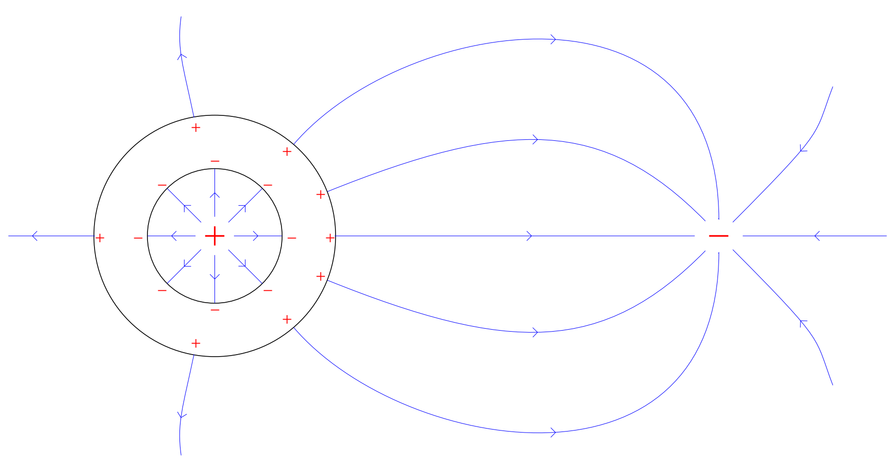
(c.) The $\vec{E}$ due to central and inner surface charges alone is shown below left. There is no net field due to these charges either *within* or *outside* the conductor.

The $\vec{E}$ due to outer surface charges alone is shown below right. There is no net field due to these charges either within the *conductor* or its *cavity*.



(d.) The *field due to the central charge* has magnitude $(1/4\pi\epsilon_0)(q/r^2)$ at all points, including within the conductor. But the *total field*, which is due to both the central and the surface charges, is zero within the conductor.

(e.) The charges within the cavity must continue to produce zero net field within or outside the conductor, so they must not move. The charges outside the conductor (i.e. the outer surface charge plus the second point charge) must produce zero net field within the conductor and cavity, so the outer surface charges must move around to shield the conductor from the second point charge.



(f.) The electric field due to the charges within the cavity is zero at the second charge, but it is attracted by the outer surface charges.

(g.) There is no force on the inside charge because it is shielded from the outside charge.

(h.) The force on the conducting shell is equal and opposite to the force on the second point charge.

Grading: 2 points each for parts (a) and (b), 1 point each for all other parts.