Oberlin College Physics 212, Fall 2021 semester

Model Solutions to Sample Final Exam

1. Sketch the seventh energy eigenfunction for the potential below.

Answer: Wavefunction will be symmetric about the origin, with six nodes, with shorter wavelength and smaller amplitude near the origin.

2. Is the nucleus ${}^{12}_{5}\mathrm{B}$ stable or unstable to beta decay? If unstable, how does it decay? Explain how you know.

Answer: See pages 22–23 of the "Notes on Nuclear and Elementary Particle Physics".

5. A free electron is said to absorb a photon. Do you believe this claim? Support your answer.

Answer: Initial, lab frame:

	E	p
electron	$m_e c^2$	0
photon	E_γ	E_γ/c
total	$m_e c^2 + E_\gamma$	E_γ/c

conserved invariant: $$E^2 - (pc)^2 = (m_e c^2 + E_\gamma)^2 - (E_\gamma)^2 = (m_e c^2)^2 + 2 m_e c^2 E_\gamma$$

Final, electron's frame:

	E	p
electron	$m_e c^2$	0

conserved invariant: $$E^2 - (pc)^2 = (m_e c^2)^2$$

Setting the two conserved invariants equal, we conclude $E_\gamma = 0$. A free electron can't absorb a photon.

6. The hydrogen atom state $|1\mathrm{s}\rangle$ has energy $-\mathrm{Ry}$, the state $|2\mathrm{p}\rangle$ has energy $-\frac{1}{4}\mathrm{Ry}$. A hydrogen atom starts off in state $\frac{4}{5}|1\mathrm{s}\rangle + \frac{3}{5}|2\mathrm{p}\rangle$. How much time elapses before the atom returns to this initial state?

Answer: This initial state evolves in time to

$$\begin{aligned}
& \tfrac{4}{5} e^{-(i/\hbar)E_{1\mathrm{s}}t}|1\mathrm{s}\rangle + \tfrac{3}{5} e^{-(i/\hbar)E_{2\mathrm{p}}t}|2\mathrm{p}\rangle \\
= \; & \tfrac{4}{5} e^{+(i/\hbar)\mathrm{Ry}\,t}|1\mathrm{s}\rangle + \tfrac{3}{5} e^{+(i/\hbar)\frac{1}{4}\mathrm{Ry}\,t}|2\mathrm{p}\rangle \\
= \; & e^{+(i/\hbar)\mathrm{Ry}\,t}\left[\tfrac{4}{5}|1\mathrm{s}\rangle + \tfrac{3}{5} e^{-(i/\hbar)\frac{3}{4}\mathrm{Ry}\,t}|2\mathrm{p}\rangle\right].
\end{aligned}$$

The exponential in front of the square brackets is a physically irrelevant global phase factor. The state comes back to the initial state whenever

$$\begin{aligned}
e^{-(i/\hbar)\frac{3}{4}\mathrm{Ry}\,t} &= 1 \\
(1/\hbar)\tfrac{3}{4}\,\mathrm{Ry}\,t &= 2\pi(\text{integer}) \\
t &= \frac{2\pi\hbar}{(3/4)\,\mathrm{Ry}}(\text{integer})
\end{aligned}$$

The shortest such time is of course $(2\pi\hbar)/(\frac{3}{4}\,\mathrm{Ry})$.

7. Electrons pass through two slits separated by 3.68 nm and result in interference maxima separated by 2.57 degrees. What was the momentum of the incoming electrons?

Strategy: The first sentence sounds like a two-slit intereference problem, but the second sentence asks about momentum. How is that supposed to fit together? We need to use the first sentence to find the electron wavelength λ, then employ the de Broglie formula $\lambda = h/p$.

Implement 1: find λ. The formula for interference maxima is $d\sin(\theta) = m\lambda$. For very small angles, like 2.57 degrees, $\sin(\theta) \approx \theta$ (in radians). Thus adjacent interference maxima separated by $\Delta\theta$ correspond to a wavelength of $d\Delta\theta = \lambda$. Converting 2.57 degrees to radians results in

$$\lambda = (3.68 \text{ nm}) \left[(2.57 \text{ deg}) \frac{\pi \text{ rad}}{180 \text{ deg}} \right] = 0.165 \text{ nm}.$$

Implement 2: find p. Use $p = h/\lambda$, so

$$p = \frac{6.626 \times 10^{-34} \text{ kg·m}^2/\text{s}}{1.65 \times 10^{-10} \text{ m}} = 4.02 \times 10^{-24} \text{ kg·m/s}.$$