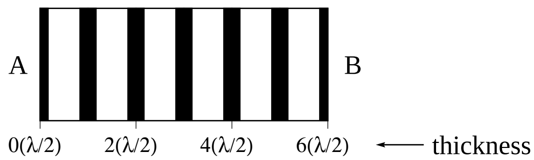
Oberlin College Physics 212, Fall 2021

Model Solutions to Assignment 5

Interferometry: Using light waves as a ruler

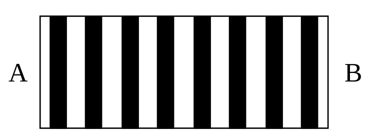
(a)



Reflection at glass-air interface: high to low, phase change no.
Reflection at air-plastic interface: low to high, phase change π.
Each bright band corresponds to an increase in thickness of $\lambda/2$.
There are six bright bands, so the thickness at right is $3\lambda = 1.8\ \mu$m.

(b)

Reflection at glass-water interface: high to low, phase change no.
Reflection at water-plastic interface: high to low, phase change no.
Thus the zero thickness band is *bright*.
Each dark band corresponds to an increase in thickness of $(\lambda/n)/2$.
The wavelength in water is not 600 nm but $\lambda/n = (600\text{ nm})/1.33 = 450$ nm.
So 1.8 μm now equals *four* wavelengths.
So there will be eight dark bands.



⟦*Grading:* 4 points for part (a); 3 points for "zero thickness band is *bright*" in part (b); 3 points for "eight dark bands" in part (b).⟧

Maxima in the single-slit diffraction intensity curve

The intensity is

$$I(\theta) = I_m \left(\frac{\sin\alpha}{\alpha} \right)^2 \qquad \text{where} \qquad \alpha = \frac{\pi a}{\lambda} \sin\theta.$$

Extrema fall where $dI/d\theta = 0$. A frontal assault on taking the derivative of

$$I(\theta) = I_m \left(\frac{\sin[(\pi a/\lambda)\sin\theta]}{[(\pi a/\lambda)\sin\theta]} \right)^2$$

would likely fail. (See "The Charge of the Light Brigade" by Alfred, Lord Tennyson: "Into the valley of death rode the six hundred.") Instead, use the chain rule: Define $z = \sin\alpha/\alpha$, and then

$$\begin{aligned}
\frac{dI}{d\theta} &= \frac{dI}{dz}\frac{dz}{d\alpha}\frac{d\alpha}{d\theta} \\
&= I_m\,(2z) \left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} \right) \left(\frac{\pi a}{\lambda}\cos\theta \right) \\
&= \frac{2 I_m \pi a}{\lambda} \left(\frac{\sin\alpha}{\alpha} \right) \left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} \right) \cos\theta.
\end{aligned}$$

An extremum falls when the above equals zero

> Either $\sin\alpha = 0$, in which case $I(\theta) = 0$ and we have a minimum,
> or else $\alpha\cos\alpha - \sin\alpha = 0$, in which case $I(\theta) \neq 0$ and we have a maximum.

Thus the condition for a maximum is

$$\alpha = \tan\alpha.$$

⟦*Grading:* 3 points for writing down intensity formula; 1 point for realizing $dI/d\theta = 0$; 3 points for taking the derivative (full credit even if $d\alpha/d\theta$ is not executed); 1 point for rejecting $\sin\alpha = 0$; 2 points for condition $\alpha = \tan\alpha$.⟧

Six solutions concerning complex arithmetic are in a separate document.