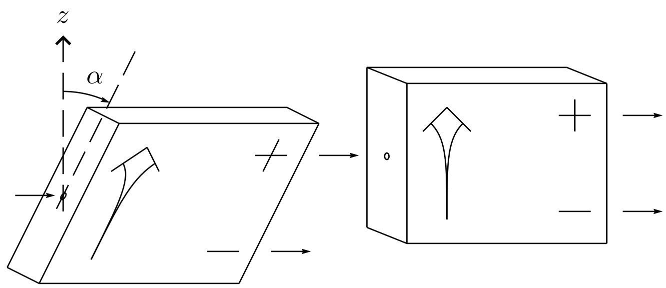
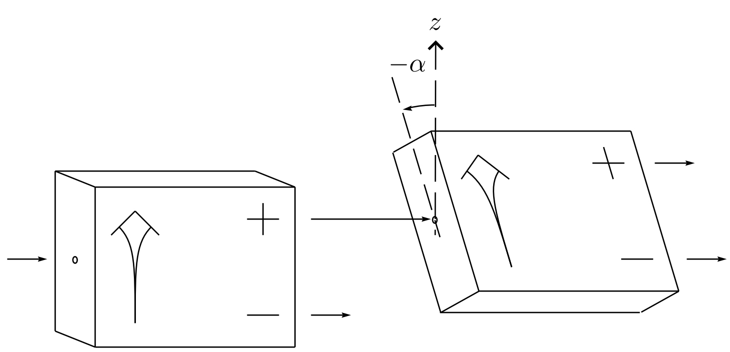
Exit probabilities

(a) Assemblage presented in assignment:



In your mind, rotate the entire assemblage by angle α counterclockwise about the axis pointing right. This will not affect the results.[1] Rotated assemblage:



After rotation the experiment is a vertical analyzer followed by an analyzer rotated from the vertical by $\theta = -\alpha$. (Or you can use $\theta = 360° - \alpha$... it's the same thing.) This is the situation of section 2.2.7. The probability of exiting from the + port is thus

$$\cos^2(-\alpha/2) = \cos^2(\alpha/2).$$

Or, if you had used $\theta = 360° - \alpha$, you would have found

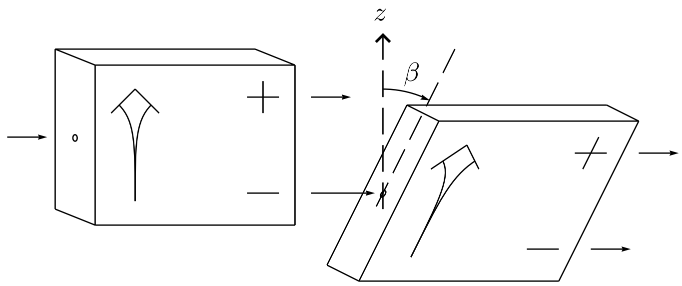
$$\cos^2((360° - \alpha)/2) = [\cos(180° - \alpha/2)]^2 = [-\cos(\alpha/2)]^2 = \cos^2(\alpha/2).$$

[As a check, note that in the special case $\alpha = 0$ this expression gives the correct probability of 1. In the special case $\alpha = 180°$, it gives the correct probability of 0. In the special case $\alpha = 90°$ (discussed in section 2.2.5), it gives the correct probability of $\frac{1}{2}$.] The probability of exiting from the − port is

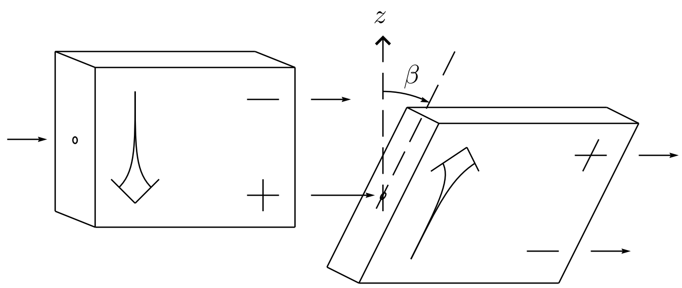
$$1 - \cos^2(\alpha/2) = \sin^2(\alpha/2).$$

[1] Alternatively and equivalently, peer down the apparatus to the right, then rotate your head *clockwise* by angle α. You *certainly* cannot affect the results by tilting your head!

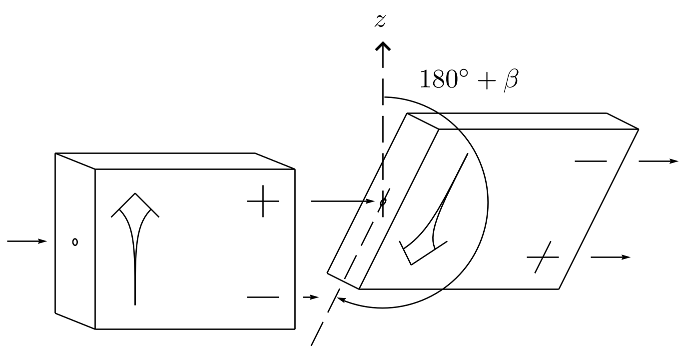
(b) Assemblage presented in assignment:



This experiment is equivalent to an upside-down analyzer, with the atom emerging from its + port fed into a β-analyzer:



In your mind, rotate the entire assemblage by 180° giving:



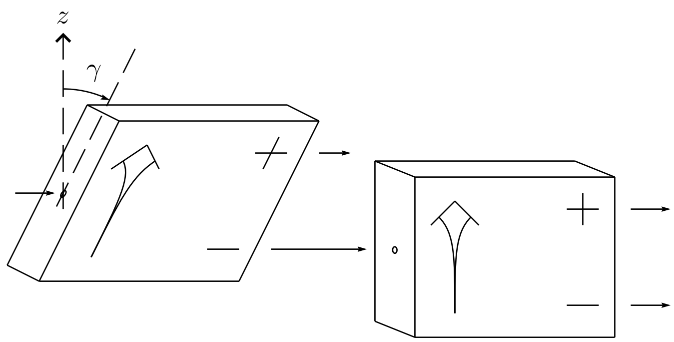
Now the experiment is a vertical analyzer followed by an analyzer rotated $\theta = 180° + \beta$ relative to the vertical. The probability of exiting from the + port is

$$\cos^2((180° + \beta)/2) = [\cos(90° + \beta/2)]^2 = [-\sin(\beta/2)]^2 = \sin^2(\beta/2).$$

The probability of exiting from the − port is

$$1 - \sin^2(\beta/2) = \cos^2(\beta/2).$$

(c) Assemblage presented in assignment:



In your mind, rotate the entire assemblage by angle γ counterclockwise about the axis pointing right. Now it is the experiment of part (b), with $\beta = -\gamma$.

From the result of part (b), the probability of exiting from the + port is

$$\sin^2(\beta/2) = \sin^2(-\gamma/2) = \sin^2(\gamma/2).$$

The probability of exiting from the − port is

$$1 - \sin^2(\gamma/2) = \cos^2(\gamma/2).$$