Oberlin College Physics 110, Fall 2011

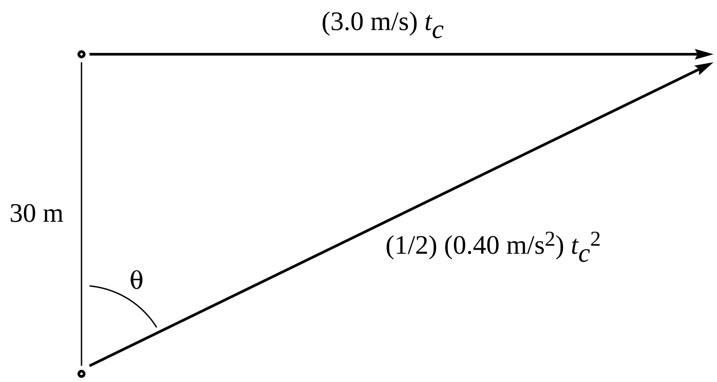
Model Solutions to Assignment 3

HRW exercise 3–16: *Vector gymnastics*

(a) $\vec{a} + \vec{b} = (8.0\text{ m})\,\hat{\text{i}} + (2.0\text{ m})\,\hat{\text{j}}$
(b) magnitude $= \sqrt{(8.0\text{ m})^2 + (2.0\text{ m})^2} = 8.2\text{ m}$
(c) angle $= \arctan(2.0/8.0) = \arctan(0.25) = 14.0°$
(d) $\vec{b} - \vec{a} = (2.0\text{ m})\,\hat{\text{i}} + (-6.0\text{ m})\,\hat{\text{j}}$
(e) magnitude $= \sqrt{(2.0\text{ m})^2 + (-6.0\text{ m})^2} = 6.3\text{ m}$
(f) angle $= \arctan(-6.0/2.0) = \arctan(-3.0) = -72°$

HRW problem 4–20: *Moving target*

At the moment of the collision (time t_c) the trajectories will look like this:



The lower angle θ is given by

$$\tan\theta = \frac{(3.0\text{ m/s})\,t_c}{30\text{ m}} = \frac{t_c}{10\text{ s}}.$$

So we can find θ if we know t_c. How do we find t_c? Through the Pythagorean Theorem!

$$(30\text{ m})^2 + (3.0\text{ m/s})^2 t_c^2 = (0.20\text{ m/s}^2)^2 t_c^4.$$

Rearrange to produce

$$(0.20\text{ m/s}^2)^2 t_c^4 - (3.0\text{ m/s})^2 t_c^2 - (30\text{ m})^2 = 0.$$

This is a quadratic equation in terms of the variable t_c^2, and using the quadratic formula $(-b \pm \sqrt{b^2 - 4ac}\,)/(2a)$ we find

$$\begin{aligned} t_c^2 &= \frac{(3.0\text{ m/s})^2 \pm \sqrt{(3.0\text{ m/s})^4 + 4(0.20\text{ m/s}^2)^2(30\text{ m})^2}}{2(0.20\text{ m/s}^2)^2} \\ &= \frac{9.0\text{ m}^2/\text{s}^2 \pm 15\text{ m}^2/\text{s}^2}{2(0.20\text{ m/s}^2)^2} \end{aligned}$$

To ensure that t_c^2 is positive, we have to take the + sign of the $\pm$. This results in

$$t_c^2 = \frac{24\ \text{m}^2/\text{s}^2}{2(0.20\ \text{m/s}^2)^2} = \frac{12\ \text{m}^2/\text{s}^2}{(0.20\ \text{m/s}^2)^2} \quad \text{or} \quad t_c = \frac{3.5\ \text{m/s}}{0.20\ \text{m/s}^2} = 17\ \text{s}.$$

Using

$$\tan\theta = \frac{t_c}{10\ \text{s}}$$

gives

$$\theta = 60^\circ.$$

Additional problem 40: *Flying apple*
Horizontal component of velocity is constant: $v_x = v_{x,0}$.
Vertical component of velocity constantly decreases: $v_y^2 = v_{y,0}^2 - 2g(y - y_0)$.
(I use the formula relating velocity and distance because we're not asked about the time.)
If the distance dropped is $-h = y_f - y_0 = -3.2$ m/s, then $v_{y,f}^2 = v_{y,0}^2 + 2gh$.
The square of the final speed is

$$v_{x,f}^2 + v_{y,f}^2 = v_{x,0}^2 + v_{y,0}^2 + 2gh$$

so

$$v_f = \sqrt{v_0^2 + 2gh} = \sqrt{(3.7\ \text{m/s})^2 + 2g(3.2\ \text{m})} = 8.7\ \text{m/s}.$$

(The launch angle is, surprisingly, irrelevant.)

Additional problem 44: *Artemis in a pickup truck*
This problem is a variation of the "range equation" (HRW 4-26). The candidates are:

(a) $(2v_T v_0^2 \sin\theta + v_0^2 \sin 2\theta)/g$

(b) $(2v_T v_0 \sin\theta + 2v_0^2 \sin 2\theta)/g$

(c) $(2v_T v_0 \cos\theta + v_0^2 \sin 2\theta)/g$

(d) $(2v_T v_0 \sin\theta + v_0^2 \sin 2\theta)/g$

These candidates suffer from the following defects:

(a) The numerator of this candidate asks us to add a term with dimensions [velocity]^3 to a term with dimensions [velocity]^2. This candidate is dimensionally inconsistent.

(b) We should recover the standard range equation (HRW 4-26) in the special case $v_T = 0$. This candidate fails this test.

(c) If $\theta = 0$ the range should be 0, but this candidate gives a range of $2v_T v_0/g$. If $\theta = 90^\circ$ the range should be positive (the arrow will come back into the truck, just as in workshop the yellow ball fell back into the cart), but this candidate gives a range of 0.

(d) There are no problems with this candidate. In fact, it is correct.

HRW problem 5–15: *Stationary salami*

In all cases, the reading of the spring scale is the tension in the cord, namely (11.0 kg)(9.81 m/s^2) = 108 N. (We assume that the masses of the cord and of the spring scale are much less than the mass of the salami.)

In case (a) the downward pull on the scale is due to the salami plus the lower cord, while the upward pull on the scale is due to the upper cord plus the ceiling.

In case (b) the rightward pull on the scale is due to the salami plus the right cord, while the leftward pull on the scale is due to the left cord plus the wall.

In case (c) the rightward pull on the scale is due to the right salami plus the right cord, while the leftward pull on the scale is due to the left cord plus the left salami.

But in all cases the two pulls sum to zero (otherwise the spring scale would accelerate), and in all cases the reading on the spring scale is the magnitude of either pull. [Comparison of cases (b) and (c) is particularly telling. The spring scale doesn't have eyes, so it can't tell whether the leftward pull is due to a wall or to a salami. By symmetry, the two cases must give the same reading.]

Note: Problem states that the spring scale is "marked in weight units". Thus the answer must be given in newtons, the unit of force, rather than kilograms, the unit of mass.

Additional problem 55: *Sliding salami*

Recall that the tension will be constant throughout the cord, and that the spring scale will read off the amount of this tension.

a. The forces on m_1 are the upward tension T from the cord and the downward pull of gravity. Taking acceleration to be positive upward gives

$$T - m_1 g = m_1 a.$$

The forces on m_2 are the tension and the pull of gravity. Taking acceleration to be positive downward gives

$$-T + m_2 g = m_2 a.$$

These are two equations in the two unknowns T and a. Solving for T gives

$$T = 2g\left(\frac{m_1 m_2}{m_1 + m_2}\right).$$

b. The equation has the required symmetry under interchange of m_1 and m_2.

c. If $m_1 = m_2$, the tension is $m_1 g$, just as it was in problem 5–15, part (c). If either m_1 or m_2 is zero, there is no tension in the cord...nothing is pulling it!

(In addition, you might want to show that the acceleration is

$$a = -g\frac{m_1 - m_2}{m_1 + m_2},$$

and to check this result for reasonableness.)