Oberlin College Physics 110, Fall 2011

Model Solutions to Sample Exam 2

Additional problem 71: *Skier.*

Key idea: Energy is conserved because poles aren't used, friction is negligible, and the normal force does no work.

Initial energy: $\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}m(0)^2 + mgh_i$.

Final energy: $\frac{1}{2}mv_f^2 + mgh_f$.

Energy conservation: $\frac{1}{2}mv_f^2 + mgh_f = mgh_i \Longrightarrow v_f^2 = 2g(h_i - h_f)$.

Change in height: $h_f - h_i = -12.3 \text{ m} + 6.5 \text{ m} - 3.5 \text{ m} + 4.4 \text{ m} - 10.7 \text{ m} = -15.6 \text{ m}$.

Final velocity: $v_f = \sqrt{2g(h_i - h_f)} = 17.5$ m/s.

Additional problem 58: *Monkey business.*

See model solutions to assignment 4.

Additional problem 80: *Ice mound, part III.*

$\frac{2}{5}R + \frac{v_0^2}{3g}$	Wrong value when $v_0 = 0$.
$\frac{2}{3}R + \frac{v_0^2}{3g}$	OK.
$\frac{2}{3}R + \frac{v_0}{3g}$	Dimensionally incorrect.
$\frac{2}{3}R - \frac{v_0^2}{3g}$	Height of departure will increase with v_0.
$\sqrt{\frac{2v_0^2 R}{3g}}$	Wrong value when $v_0 = 0$.

Additional problem 66: *Simple harmonic motion graph, II.*

a. We have

$$\begin{aligned} x(t) &= A\sin(\omega t) \\ v(t) &= A\omega\cos(\omega t) \\ a(t) &= -A\omega^2\sin(\omega t). \end{aligned}$$

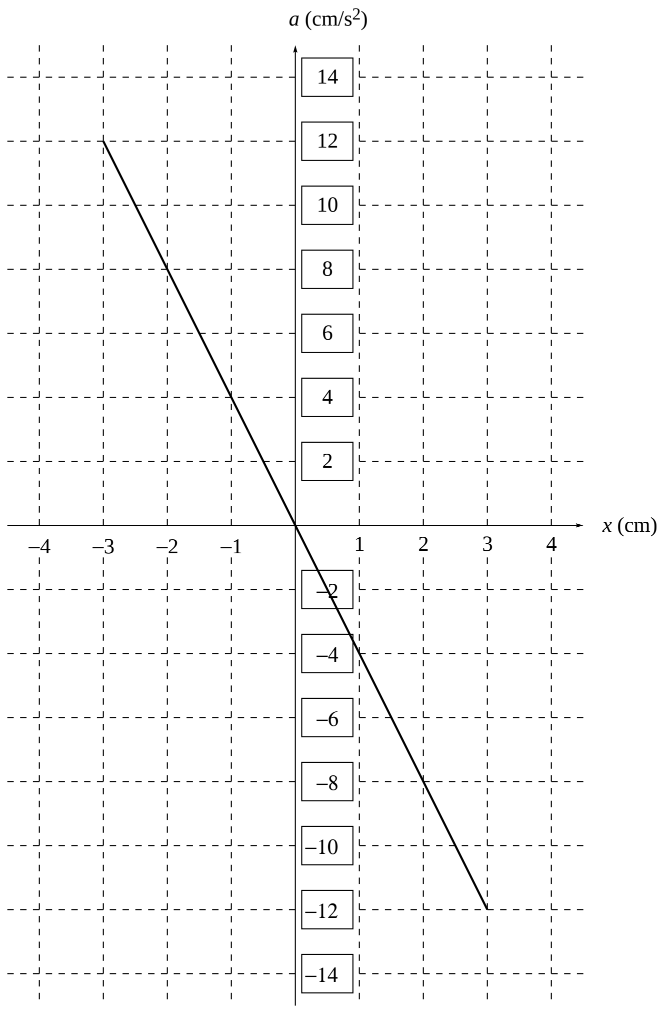
Because $\sin\theta$ varies between $+1$ and -1, the maximum acceleration will come when the sine is equal to -1, namely

$$\text{maximum acceleration} = A\omega^2 = A\left(\frac{2\pi}{T}\right)^2 = 12.0 \text{ cm/s}^2.$$

b. Comparing $a(t)$ and $x(t)$ above, $a(t) = -\omega^2 x(t)$. Alternatively,

$$a = \frac{F}{m} \quad \text{but} \quad F = -kx \quad \text{so} \quad a = -\frac{k}{m}x.$$

The graph is thus



Additional problem 70: *Work with force preconceptions.*

In truth $\sum \vec{F} = m\vec{a}$, from which we proved in class that (where i stands for "initial" and f stands for "final")

$$\begin{aligned} \sum \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} &= m \int_{\vec{r}_i}^{\vec{r}_f} \vec{a} \cdot d\vec{r} \\ \sum W &= m \int_{t_i}^{t_f} \frac{d\vec{v}}{dt} \cdot \frac{d\vec{r}}{dt}\, dt \\ &= m \int_{t_i}^{t_f} \frac{d\vec{v}}{dt} \cdot \vec{v}\, dt \\ &= \tfrac{1}{2} m \int_{t_i}^{t_f} \frac{d\vec{v}^2}{dt}\, dt \\ &= \tfrac{1}{2} m \left[\vec{v}^2\right]_{t_i}^{t_f}. \end{aligned}$$

⟦The above is not needed to solve the problem. I put it in only to facilitate comparison between what happens in truth and what would happen if the preconception were true.⟧

But if you held the pseudo-Aristotelian preconception that $\sum \vec{F} = m\vec{v}$, then you would have to hold that

$$\begin{aligned} \sum \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} &= m \int_{\vec{r}_i}^{\vec{r}_f} \vec{v} \cdot d\vec{r} \\ \sum W &= m \int_{t_i}^{t_f} \vec{v} \cdot \frac{d\vec{r}}{dt}\, dt \\ &= m \int_{t_i}^{t_f} \vec{v}^2\, dt, \end{aligned}$$

and the right-hand side is always positive or zero.

Additional problem 89: *Impulse with force preconceptions.*

In truth $\sum \vec{F} = m\vec{a}$, from which we proved in class that (where i stands for "initial" and f stands for "final")

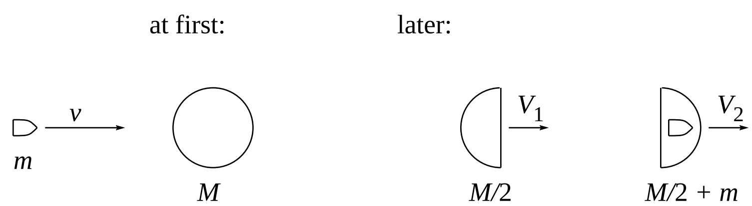
$$\begin{aligned} \sum \int_{t_i}^{t_f} \vec{F}\, dt &= m \int_{t_i}^{t_f} \vec{a}\, dt \\ \sum \vec{J} &= m \int_{t_i}^{t_f} \frac{d\vec{v}}{dt}\, dt \\ &= m \left[\vec{v}\right]_{t_i}^{t_f}. \end{aligned}$$

⟦The above is not needed to solve the problem. I put it in only to facilitate comparison between what happens in truth and what would happen if the preconception were true.⟧

But if you held the pseudo-Aristotelian preconception that $\sum \vec{F} = m\vec{v}$, then you would have to hold that

$$\begin{aligned} \sum \int_{t_i}^{t_f} \vec{F}\, dt &= m \int_{t_i}^{t_f} \vec{v}\, dt \\ \sum \vec{J} &= m \int_{t_i}^{t_f} \frac{d\vec{r}}{dt}\, dt \\ &= m \left[\vec{r}\right]_{t_i}^{t_f} . \end{aligned}$$

7. *Cracked up cantaloupe.*



momentum: $mv = \frac{M}{2}V_1 + \left(\frac{M}{2} + m\right) V_2$

kinetic energy: $\frac{1}{2}mv^2 = \frac{1}{2}\frac{M}{2}V_1^2 + \frac{1}{2}\left(\frac{M}{2} + m\right) V_2^2$

Because $M/2$ is about 500 times m, approximate $M/2 + m$ by $M/2$.

from momentum: $v = \frac{M}{2m}(V_1 + V_2)$

from kinetic energy: $v^2 = \frac{M}{2m}(V_1^2 + V_2^2)$

square of "from momentum": $v^2 = \left(\frac{M}{2m}\right)^2 (V_1^2 + 2V_1V_2 + V_2^2)$

equating the two expressions for v^2 gives:

$$\frac{M}{2m}(V_1^2 + V_2^2) = \left(\frac{M}{2m}\right)^2 (V_1^2 + 2V_1V_2 + V_2^2)$$

$$V_1^2 + V_2^2 = \frac{M}{2m}(V_1^2 + 2V_1V_2 + V_2^2)$$

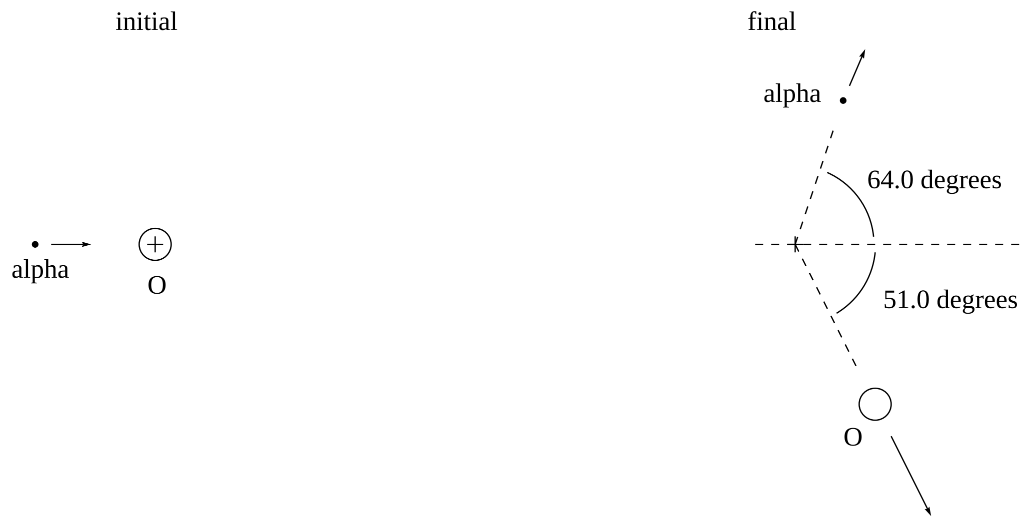
Remembering that $M/2m$ is about 500, this is

$$0 \approx \frac{M}{2m}(V_1^2 + 2V_1V_2 + V_2^2)$$

$$2V_1V_2 \approx -(V_1^2 + V_2^2)$$

The right hand side is negative, so V_1 and V_2 are of opposite sign: they fly away in different directions.

8. *Atomic collision.*



Conservation of momentum:

$$\text{Horizontal:} \quad m_\alpha v_{\alpha,i} = m_\alpha v_{\alpha,f}\cos(64.0^\circ) + m_\mathrm{O} v_{\mathrm{O},f}\cos(51.0^\circ) \tag{1}$$

$$\text{Vertical:} \quad 0 = m_\alpha v_{\alpha,f}\sin(64.0^\circ) - m_\mathrm{O} v_{\mathrm{O},f}\sin(51.0^\circ) \tag{2}$$

We know the speed of the outgoing nucleus, $v_{\mathrm{O},f} = 1.20 \times 10^5$ m/s, and the two masses, so these are two equations in two unknowns. Equation (2) tells us that

$$v_{\alpha,f} = v_{\mathrm{O},f}\frac{m_\mathrm{O}}{m_\alpha}\frac{\sin(51.0^\circ)}{\sin(64.0^\circ)} = 4.15 \times 10^5 \text{ m/s},$$

while equation (1) tells us that

$$v_{\alpha,i} = v_{\alpha,f}\cos(64.0^\circ) + \frac{m_\mathrm{O}}{m_\alpha} v_{\mathrm{O},f}\cos(51.0^\circ) = 4.84 \times 10^5 \text{ m/s}.$$