Visualizing Poynting vector energy flow in electric circuits

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Abstract

According to standard Poynting vector arguments, energy in a circuit flows from the batteries to the resistors, not through the connecting wires, but through empty space between the wires. The computer simulation `CircuitSurveyor` helps to visualize this counterintuitive fact. The simulation also demonstrates the electric fields present near a circuit.

I. THE PURPOSE

When a battery is connected to a resistor by wires to form a circuit, current flows through the wires. In addition, electromagnetic energy flows from the battery to the resistor (the battery discharges and the resistor warms up). It is natural to surmise that the energy, like the charge, flows through the wires.

According to Poynting vector theory[1] this surmise, although perfectly natural, is incorrect. Electromagnetic energy is located throughout space with energy density

$$u(\mathbf{r},t) = \frac{1}{2}\left(\epsilon_0 E^2(\mathbf{r},t) + \frac{1}{\mu_0}B^2(\mathbf{r},t)\right), \tag{1}$$

and flows with power per cross-sectional area equal to the Poynting vector

$$\mathbf{S}(\mathbf{r},t) = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}. \tag{2}$$

Ideal wires have zero resistivity ρ and hence, by Ohm's law $\mathbf{E} = \rho\mathbf{J}$, zero electric field as well. Instead of transporting power, the power flow within any ideal wire is exactly zero!

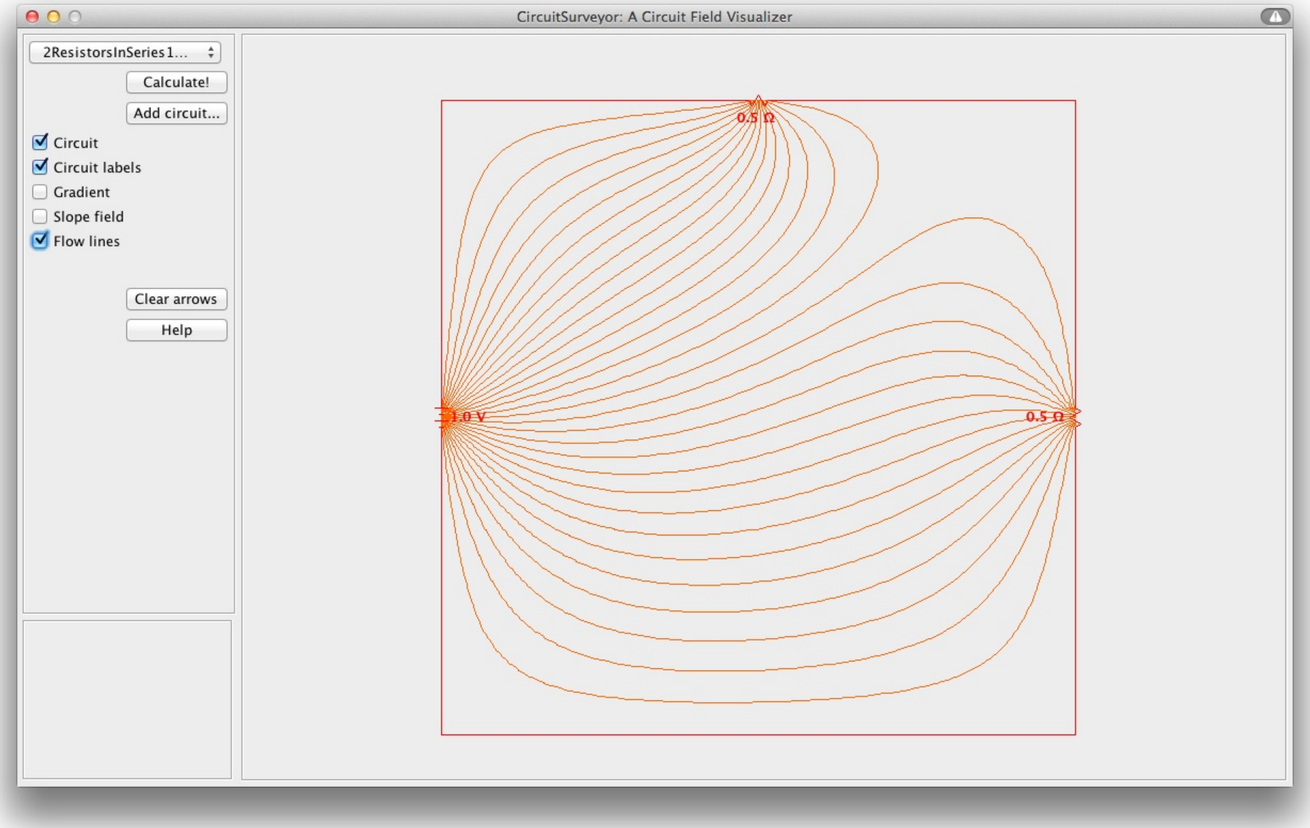
Many students find these results difficult to accept. If wires do not transmit power, then why do power companies go to enormous expense to put up transmission wires? Arguments effective in making these points include:

- The drift velocity of charge carriers is constant throughout a single-loop circuit of constant wire diameter, so the energy cannot be carried in the kinetic energy of the charge carriers.
- The potential energy cannot be successfully assigned to individual charge carriers, but either (1) to pairs of charges or, better, (2) to fields.[2]
- Energy cannot be transported by the charge carriers because in alternating current circuits, there is no net charge transport, but there certainly *is* net energy transport.
- If energy is conserved in all reference frames, it cannot just disappear at the battery and reappear simultaneously at the resistor, because (according to special relativity) the disappearance and reappearance would be simultaneous in one reference frame, but in other frames the disappearance would happen first and the reappearance happen after a time lag, and in yet other frames the reappearance would happen before the disappearance had yet occurred!

- It is clear that electromagnetic energy can flow from the Sun to the Earth without passing through wires. Why not also from the battery to the resistor?
- In a transformer (or any mutual inductor), energy flows from primary to secondary coils without passing through wire.
- The Feynman disk paradox[3] demonstrates that even static electromagnetic fields must (in some circumstances) possess momentum. If fields have momentum, it is a small jump to think that they have energy as well.

All of these strategies are good, but all of them appeal to the intellect and not to any intuitive "gut-feeling" for the physics. The wires are clear and visual and appealing. Is there any way to make the energy flow through space visual and appealing?

There is. Inspired by the work of Majcen, Haaland, and Dudley,[4] we have developed a computer simulation to trace Poynting vector energy flows in circuits. The program, `CircuitSurveyor`[5], takes a two-dimensional circuit as input, and then finds the associated electric, magnetic, and Poynting fields.



2ResistorsInSeries1...

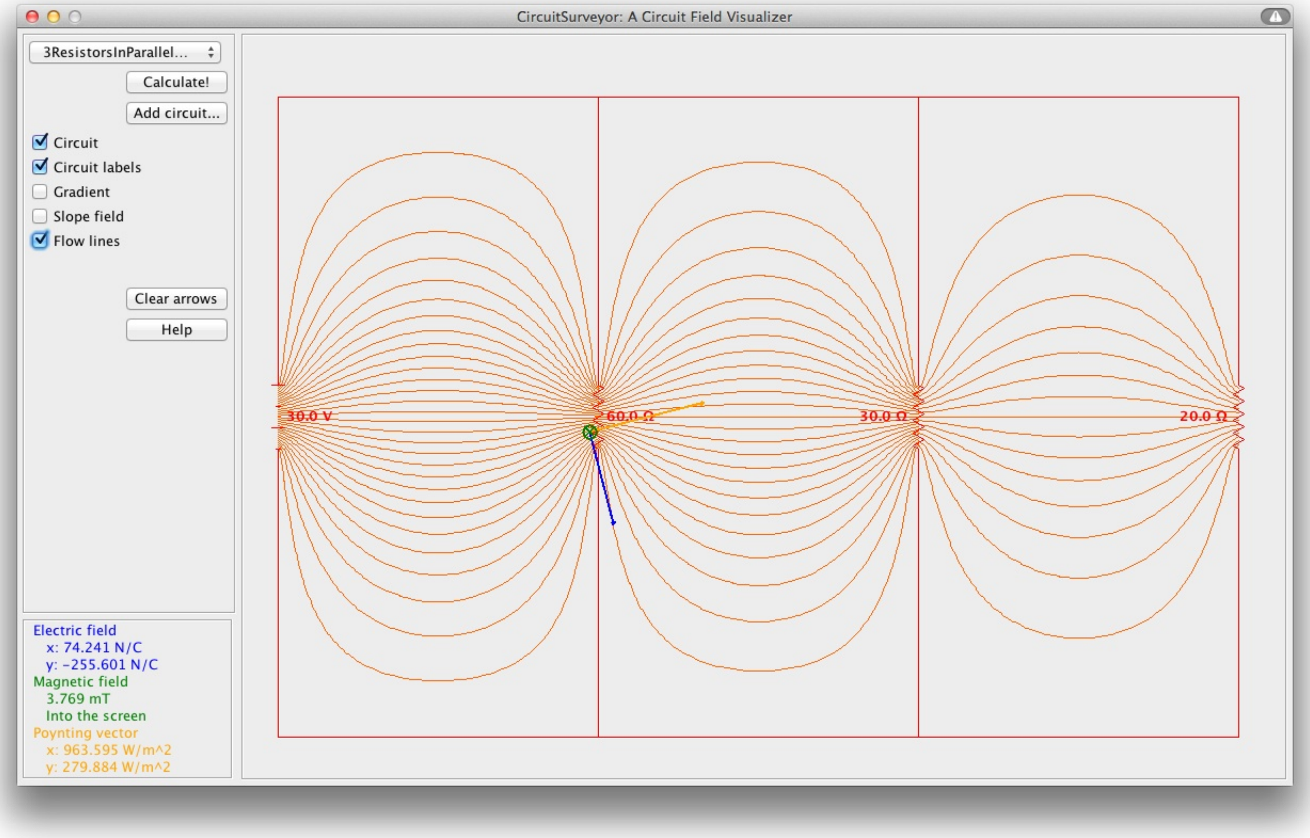
☑ Circuit
☑ Circuit labels
☐ Gradient
☐ Slope field
☑ Flow lines

Figure 1: Poynting flow lines for a circuit with one 1.0 V battery and two 0.5 Ω resistors in series. The energy flows through space from the battery and arrives in equal amounts at each of the two resistors (15 flow lines each). (This is the circuit investigated in Ref. 4.)

Although the primary role of `CircuitSurveyor` is to trace the Poynting energy flow using flow lines (similar to flow lines in moving water, or to electric field lines in electrostatics), it also finds the electric field. Hence it can be used to demonstrate field aspects of circuits, in contrast to the predominant emphasis on potentials.[6–12]

II. THE MODEL AND THE ALGORITHM

`CircuitSurveyor` simulates two-dimensional circuits, such as those shown in Figs. 1 and 2. By this we do *not* mean a circuit lying flat on a lab bench in a three-dimensional world. Instead we mean that the circuit is a perpendicular slice through a stack of identical circuits extending infinitely into and out of the computer display. Only by using this model is the analysis tractable and the visualization (on a two-dimensional display) effective.



☑ Circuit
☑ Circuit labels
☐ Gradient
☐ Slope field
☑ Flow lines

Electric field
x: 74.241 N/C
y: −255.601 N/C
Magnetic field
3.769 mT
Into the screen
Poynting vector
x: 963.595 W/m^2
y: 279.884 W/m^2

Figure 2: Poynting flow lines for a circuit with one 30.0 V battery and three parallel resistors of resistance (from left to right) of 60.0 Ω, 30.0 Ω, and 20.0 Ω. Power dissipation in these three resistors is 15.0 W, 30.0 W, and 45.0 W, respectively. And, sure enought, of the 30 Poynting flow lines leaving the battery, 5 terminate at the first resistor, 10 at the second, and 15 at the third. In addition, each resistor is 10.0 cm long and supports a potential drop of 30.0 V, whence we expect an electric field of about 300 N/C near each resistor. Sure enough, the computed electric field at the point shown is 266 N/C.

To find the Poynting vector one must start by finding the electric and magnetic fields at all points. The program first finds the current in each leg of wire, and the potential drop across each resistor, using standard Kirchhoff methods. Once this is done we know, for every rectilinear polygon within the circuit (such as the rectangular areas in Figs. 1 and 2), the electrostatic potential at the edge of that polygon. From this edge information we use a relaxation method[13] to find the potential within the polygon, and from the potential the electric field.

Finding the magnetic field is even simpler. In this model each rectilinear polygon of wire on the display represents a slice through an infinitely long solenoid of constant cross-sectional size and shape, and the magnetic field inside such a solenoid is perpendicular to the plane of the display with magnitude $\mu_0 i/d$ (where μ_0 represents the vacuum magnetic permeability, i the current, and d the diameter of an individual wire). The magnetic field outside such a solenoid is zero, so there is no power flow in the space outside the circuit.

Because the magnetic field is perpendicular to the plane, the Poynting vector lies within the plane and perpendicular to the electric field. Thus the Poynting vector points along equipotentials, or in other words the Poynting flow lines are equipotentials.

The two-dimensional model is necessary for both fast computation and effective visualization. However, it does introduce an artifice. In a three-dimensional circuit a resistor has an infinite number of sides and power flows in through all of them.[1] In contrast, in this two-dimensional model, a resistor has only two sides and power flows in only from the inside of the circuit.

III. THE SIMULATION

The simulation `Circuit Surveyor` provides ten built-in circuits, or users may design their own custom circuits (instructions are provided). Moving the mouse pointer into the circuit will draw a triplet of arrows showing the electric field, the magnetic field, and the Poynting vector at that point — the numerical values of these vectors are also output (see Fig. 2). A mouse click at any point within the circuit fixes the drawn arrows. Multiple clicks thus enable the user to sketch out and obtain a qualitative feel for the electric, magnetic, and Poynting fields.

The quantitative results produced are for wire of thickness 1 mm. In Fig. 1, the circuit is 100 cm by 100 cm, and the battery and two resistors are each 3 cm long. In Fig. 2 the circuit is 100 cm by 150 cm, and each element is 10 cm long.

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Edwin Taylor and Bruce Sherwood critiqued various versions of this paper and the computer simulation. Oberlin College student Henrietta Onumah did exploratory thinking concerning this project in the Fall 2000 semester. Two anonymous referees suggested improvements. This work was supported by the John and Marianne Schiffer Professorship in Physics and by Oberlin College.

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[1] David J. Griffiths, *Introduction to Electrodynamics*, 3rd ed. (Prentice Hall, Upper Saddle River, NJ, 1999), pp. 346–349.

[2] Reference 1, p. 96.

[3] Richard P. Feynman, Robert B. Leighton, and Matthew Sands, *The Feynman Lectures on Physics*, Vol. II (Addison-Wesley, Reading, MA, 1964), pp. 17-6 and 27-11.

[4] Slavko Majcen, Ryan K. Haaland, and Scott C. Dudley, "The Poynting vector and power in a simple circuit," Am. J. Phys. **68**, 857–859 (2000).

[5] The program can be run or downloaded from

`<www.oberlin.edu/physics/dstyer/CircuitSurveyor>`,

and is also available from EPAPS at

`<http://dx.doi.org/10.1119/1.3679838>`.

The Java source code and executable are freely available and are released to the public under the terms of the GNU General Public License, version 3. The program runs under Java SE 6 (internal version number 1.6) or higher. `CircuitSurveyor` is distributed without warranty. Despite our best efforts to make it portable, it cannot be guaranteed to run with every version of Java under every operating system on every computer. To use it locally, download `CircuitSurveyor.zip` and unzip it into any directory, for example into `MyPhysicsPrograms/EMPrograms/CircuitSurveyor`. Use a text editor to change the phrase

`http://www.oberlin.edu/physics/dstyer/CircuitSurveyor/`

in `LaunchCircuitSurveyor.jnlp` to, for example,

`file://MyPhysicsPrograms/EMPrograms/CircuitSurveyor/`.

Then double click on the edited file `LaunchCircuitSurveyor.jnlp`.

[6] Mark A. Heald, "Electric fields and charges in elementary circuits," Am. J. Phys. **52**, 522–526 (1984).

[7] J. D. Jackson, "Surface charges on circuit wires and resistors play three roles," Am. J. Phys. **64**, 855–870 (1996).

[8] Norris W. Preyer, "Surface charges and fields of simple circuits," Am. J. Phys. **68**, 1002–1006 (2000).

[9] Igal Galilia and Elisabetta Goihbarg, "Energy transfer in electrical circuits: A qualitative account," Am. J. Phys. **73**, 141–144 (2005).

[10] Manoj K. Harbola, "Energy flow from a battery to other circuit elements: Role of surface charges," Am. J. Phys. **78**, 1203–1206 (2010).

[11] Rebecca Jacobs, Alex de Salazar, and Antonio Nassar, "New experimental method of visualizing the electric field due to surface charges on circuit elements," Am. J. Phys. **78**, 1432–1433 (2010).

[12] Ruth Chabay and Bruce Sherwood, *Matter and Interactions*, 3rd ed., Vol. II (John Wiley & Sons, New York, 2011), Chap. 19.

[13] Richard V. Southwell, *Relaxation Methods in Theoretical Physics* (Oxford University Press, Oxford, UK, 1946); David A. Hastings, "Computational method for electrical potential and other field problems," Am. J. Phys. **43**, 518–524 (1975).