Vector calculus

The **divergence theorem** (Gauss's theorem):

If the volume $\mathcal{V}$ is enclosed by surface $\mathcal{S}$, then

$$\int_{\mathcal{V}} \vec{\nabla} \cdot \vec{F}(\vec{r})\, d^3r = \int_{\mathcal{S} \text{ of } \mathcal{V}} \vec{F}(\vec{r}) \cdot \hat{n}\, dA.$$

You may call this a theorem if you wish, but to me it just pops right out of the definition

$$\vec{\nabla} \cdot \vec{F}(\vec{r}_0) = \lim \frac{\int_{\mathcal{S} \text{ of } \mathcal{V}} \vec{F}(\vec{r}) \cdot \hat{n}\, dA}{\text{volume of } \mathcal{V}}$$

where the limit goes through any sequence of volumes closing in around point $\vec{r}_0$.

The **circulation theorem** (Stokes's theorem):

If the surface $\mathcal{S}$ is bounded by edge $\mathcal{E}$, then

$$\int_{\mathcal{S}} (\vec{\nabla} \times \vec{F}(\vec{r})) \cdot \hat{n}\, dA = \int_{\mathcal{E} \text{ of } \mathcal{S}} \vec{F}(\vec{r}) \cdot d\vec{\ell}.$$

To me, this just pops right out of the definition

$$(\vec{\nabla} \times \vec{F}(\vec{r}_0)) \cdot \hat{n} = \lim \frac{\int_{\mathcal{E} \text{ of } \mathcal{S}} \vec{F}(\vec{r}) \cdot d\vec{\ell}}{\text{area of } \mathcal{S}}$$

where the limit goes through any sequence of surfaces perpendicular to $\hat{n}$ and closing in around point $\vec{r}_0$.