

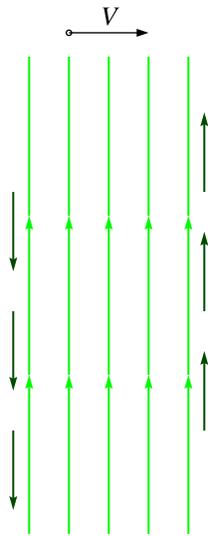
A changing magnetic field makes an electric field. A changing electric field makes a magnetic field.

These phenomena are expressed mathematically through the Faraday law and the Ampere-Maxwell law:

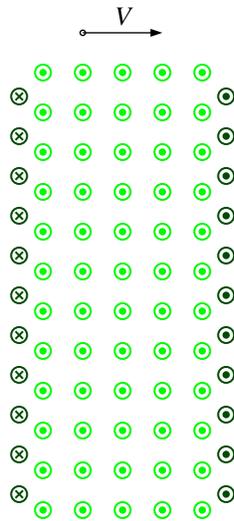
$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \quad \text{and} \quad \oint \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \quad (1)$$

As an example, consider an infinite slab of uniform magnetic field moving right at velocity  $V$ . (For now, don't ask what the source of this  $\vec{B}$  is.) If you imagine standing still as this slab of  $\vec{B}$  washes over you, you will realize that the front of the slab is accompanied by  $d\vec{B}/dt$  pointing upward, and that the rear is accompanied by  $d\vec{B}/dt$  pointing downward. (As usual,  $\vec{B}$  is represented by green, and  $d\vec{B}/dt$  by dirty green.)

Side View

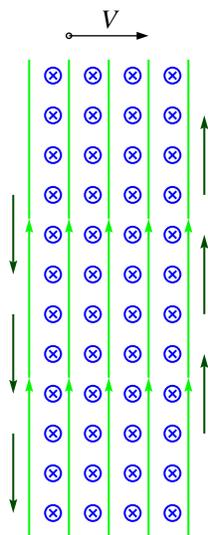


Top View

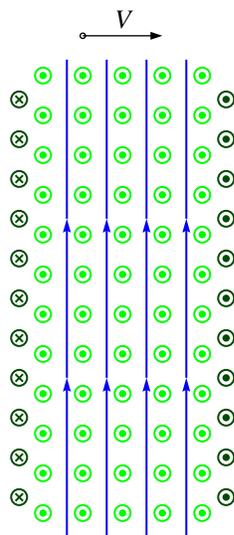


As Faraday discovered, this  $d\vec{B}/dt$  is the source of  $\vec{E}$ , which in this case is internal to the slab. Its qualitative character can be determined (compare the homework problem of finding  $\vec{B}$  due to “infinite sheets of current”) by the “anti-right-hand rule”, which shows an electric field as sketched here (as usual,  $\vec{E}$  is represented by blue):

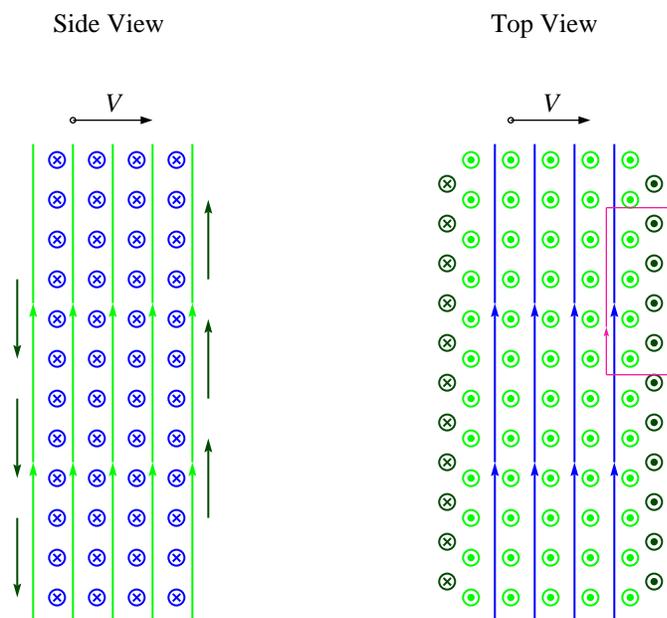
Side View



Top View



To find this field's magnitude, apply Faraday's law to the purple loop below:



We find

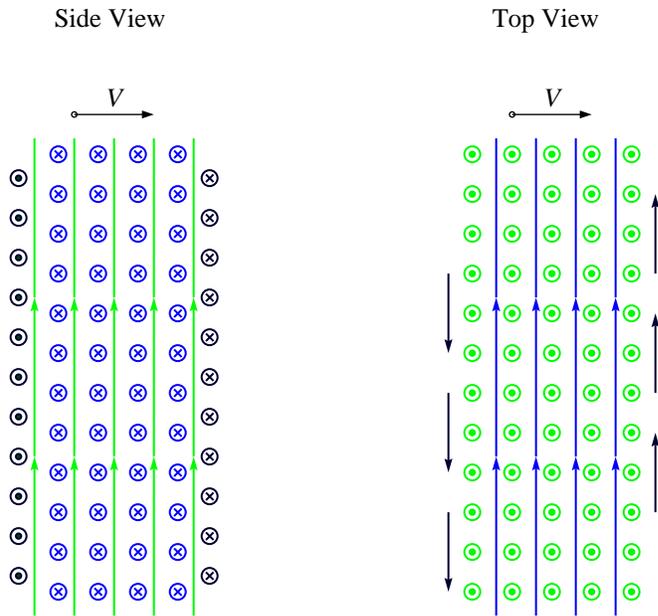
$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$|\vec{E}|(\text{height of loop}) = |\vec{B}|V(\text{height of loop})$$

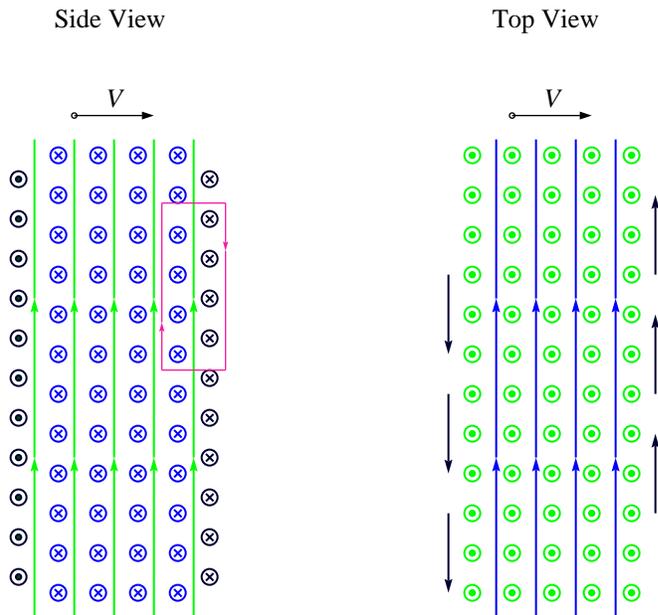
$$|\vec{E}| = |\vec{B}|V \tag{2}$$

(This makes perfectly good sense: the  $\vec{E}$  is caused by changing  $\vec{B}$ , so if there's more magnitude of  $\vec{B}$ , there's more magnitude of  $\vec{E}$ , and if there's more change through a larger  $V$ , there's again more magnitude of  $\vec{E}$ .)

Now of course, this moving slab of  $\vec{E}$  has a  $d\vec{E}/dt$  at its front and rear edges (the figure below shows the  $d\vec{E}/dt$  — represented in dirty blue — but to avoid clutter it doesn't show the  $d\vec{B}/dt$ ):



As Maxwell showed, this  $d\vec{E}/dt$  generates magnetic field. This is the source of the original  $\vec{B}$ ! To find its magnitude, apply the Ampere-Maxwell law to the purple loop below:



We find

$$\begin{aligned}\oint \vec{B} \cdot d\vec{\ell} &= \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \\ |\vec{B}|(\text{height of loop}) &= \mu_0 \epsilon_0 |\vec{E}| V(\text{height of loop}) \\ |\vec{B}| &= \mu_0 \epsilon_0 |\vec{E}| V\end{aligned}\tag{3}$$

Putting together equations (2) and (3), we discover that

$$V = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.\tag{4}$$