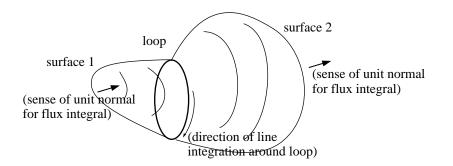
Apparent oversight in the flux rule

Griffiths, Electrodynamics, fourth edition, problem 7.9



Here's the loop. (The unit normals are drawn in accord with the right hand rule described by Griffiths on the top of page 298.) Does Φ_B refer to the flux through surface 1 ($\Phi_{B,1}$) or the flux through surface 2 ($\Phi_{B,2}$)?

Surface 1 plus surface 2 combined to make a *closed* surface, which I'll call surface T (for "total"). If you calculate the (outward) flux of \vec{B} through surface T, you'll find zero — this is Gauss's law of magnetism:

$$\Phi_{B,T} = 0.$$

But when finding the outward flux, we use an outward unit normal everywhere. This is the same as the unit normal adopted above for surface 2 but the negative of the unit normal adopted above for surface 1. Thus

$$\Phi_{B,T} = -\Phi_{B,1} + \Phi_{B,2}.$$

Our two results combine to tell us that

$$\Phi_{B,1} = \Phi_{B,2}.$$

It doesn't care which surface we use for the flux rule.