Assignment #4

PHYS-410
Fall 2013
Mr. Scofield

Reading

Topics this week (in Ch. 6) include density of states and Maxwell-Boltzmann distribution. Please review the notes on Density of States that I have posted on the class web page.

Also, read Chapter 3, Sections 1-3. The first two sections provide insights into issues we have been talking about this last few lectures. Pay particular attention to Section 3.3 on Paramagnetism. Also see pages 144-147 about reaching very low temperatures using adiabatic demagnetization. Important concepts include: Curie's Law, negative temperature, and population inversion.

Homework Problems (from Schroeder, unless otherwise specified)

Your solutions to the problems below are due at the beginning of class, Friday, October 4.

4.01 Consider a system in thermal contact with a temperature-bath at temperature T. Let C_V be the constant-volume heat capacity of the system. Show that the variance in the energy U is given by

$$\overline{\left(U - \overline{U}\right)^2} = k_B T^2 C_V \,.$$

(Note, this is essentially Problem 6.18.)

4.02 A very sensitive spring balance consists of a quartz spring suspended from a fixed support. The spring constant is K. That balance is at a temperature T in a location where the acceleration due to gravity is g.

a) If a small object of mass m is suspended from the spring, what is the mean resultant elongation $\bar{x}$ of the spring?

b) What is the magnitude $\overline{(x-\bar{x})^2}$ of the thermal fluctuations of the object about its equilibrium position?

c) If the fluctuations are so large that $\overline{(x-\bar{x})^2} \approx \bar{x}^2$ it becomes impractical to measure the mass of the object. What is the minimum mass m that can be measured with this balance?

4.03 Consider a monatomic "gas" consisting of N, non-interacting atoms, each to be considered only as point particles of mass m, confined to a 3D infinite cubic well of volume $V = L^3$.

a) Use the density of states $D(\varepsilon)$ to find the partition function for a single particle, $Z_1(T,V) = \int_0^\infty D(\varepsilon) e^{-\beta\varepsilon} d\varepsilon$.

b) What is the partition function $Z(T,V,N)$ for the N-particle system? (Hint: see the discussion in section 6.7.)

c) Find the mean energy $\overline{U} = -\dfrac{\partial \log Z(\beta,V,N)}{\partial\beta}$.

d) Find the heat capacity, C_V, for this system.

e) Find the pressure from $\beta p = \dfrac{\partial \log Z(\beta,V,N)}{\partial V}$.

Show that your result is equivalent to the ideal gas law, $pV = Nk_B T$.

4.04 Generalize the argument presented in class for the particle in the 3-D infinite square well to find the density of states for 1- and 2-D.

4.05 Consider the Maxwell speed distribution, namely

$$P(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_B T}.$$

(a) By direct differentiation show that the distribution peaks at a speed

$$v_p = \sqrt{2k_B T/m}.$$

(b) Perform the necessary integral to show that the mean speed is given by

$$\bar{v} \equiv \int_0^\infty vP(v)dv = \sqrt{8k_B T/\pi m}.$$

(c) Finally, perform the necessary integration to show that the rms-speed is given by

$$v_{rms}^2 \equiv \int_0^\infty v^2 P(v)dv = 3k_B T/m.$$

(Note: these are essentially Problems 6.35, 6.36, and 6.37.)

4.06 Due to concerns over global climate change there is great interest in finding ways to reduce GHG emission by, say, capturing and storing CO_2 from the flue gases of coal plants. Some are even considering the possibility of extracting CO_2 from the ambient atmosphere – a technology that could be used anywhere. It is reasonable to inquire about the minimum energy required to extract and store CO_2 by either of these two methods. Once captured the CO_2 would be compressed to, say, 100 atmospheres and stored deep underground. In this problem you are to put thermodynamic limits on the required energy.

Consider a container of volume V at temperature T = 300 K that is filled with two ideal gases – nitrogen at atmospheric pressure p_0 and CO_2 with a partial pressure p_C, $p_C << p_0$. Suppose that V is sufficiently large that it contains exactly 1 mole of CO_2. Calculate the energy required to separate the gases and compress the CO_2 to pressure p_0 (final temperature 300 K). You may treat both gases as "ideal." Be sure to state any assumptions you are making.

For both flue gas ($p_C/p_0 \approx 10\%$) and ambient air ($p_C/p_0 \approx 350x10^{-6}$) calculate the energy per mole required to extract CO_2 and compress it to 100 atmospheres. Obtain numerical values and compare with the heat released from burning coal, roughly 500 kJ per mole CO_2 produced. Comment on the kind of energy required (heat or work) to accomplish this capture and storage.

Hint: There are several ways to approach this problem. One is to insert a "semi-permeable" wall at one end of the container (imagine a rectangular volume) through which one gas passes freely and the other is blocked. You can then slowly move this semi-permeable wall to compress one of the gases and not the other. Ideally such walls can be inserted and removed without any energy.

Alternately you might imagine the reverse process. Imagine a small balloon (volume V_C) filled with 1 mole of CO_2 inside the larger container (volume V) of nitrogen at atmospheric pressure. If you "pop" the balloon you fill the entire volume V with a low partial pressure of CO_2.