Assignment #2

PHYS-410
Fall 2013
Mr. Scofield

Announcements

1. The solutions to your first homework assignment have been distributed in class.

Reading

We now begin our study of *Statistical Mechanics*.

Reach Chapter 2 of Schroeder. All sections are important. Here is a brief list of the important concepts and topics: multiplicity, combinational formula $\binom{N}{n}$, Einstein model, fundamental axiom of statistical mechanics, second law of thermodynamics, entropy, Stirling's approximation $N! \approx N^N e^{-N}\sqrt{2\pi N}$, ideal gas, entropy of mixing.

I don't like Schroeder's treatment of the ideal gas and plan to put off our discussion of the ideal gas until later in the course, when I can use more sophisticated tools to approach the problem – more later.

Homework Problems (from Schroeder, unless otherwise specified)

Your solutions to the problems below are due at the beginning of class, Friday, Sept. 20.

<u>2.01</u>

4.1 (*a*) One kilogram of water at 0°C is brought into contact with a large heat reservoir at 100°C. When the water has reached 100°C, what has been the change in entropy of the water? of the heat reservoir? of the entire system consisting of both water and heat reservoir?

(*b*) If the water had been heated from 0°C to 100°C by first bringing it in contact with a reservoir at 50°C and then with a reservoir at 100°C, what would have been the change in entropy of the entire system?

(*c*) Show how the water might be heated from 0°C to 100°C with no change in the entropy of the entire system.

(above problem borrowed from Reif.)

<u>2.02</u> **Rosser Fig. 1.4**

As another example of an irreversible process, consider two subsystems of equal volumes V, as shown in Figure 1.4(a). Subsystem 1 consists of a mole of ideal gas molecules, whereas subsystem 2 is a vacuum. If the partition in Figure 1.4(a) is removed, the gas expands to fill the whole volume $2V$, as shown in Figure 1.4(b). This is an irreversible process, since,

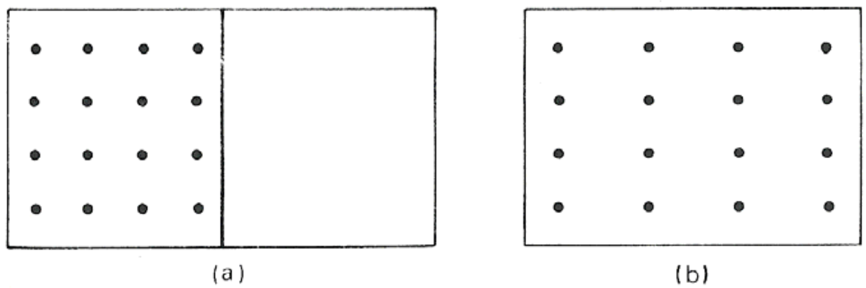

Figure 1.4—Example of an irreversible process. When the partition in (a) is removed the ideal gas expands from a volume V to occupy a volume $2V$. The entropy of the gas increases.

after the partition has been removed, one would never find all the gas molecules back in the left hand side half of the composite system in Figure 1.4(b). It can be shown using classical thermodynamics or using statistical mechanics (Reference: Problem 5.9 of Chapter 5) that, in the example illustrated in Figures 1.4(a) and 1.4(b), the increase in the entropy per mole is

$$\Delta S = R \log 2$$

where log 2 is the natural logarithm of 2 to the base e.

Note – find the solution to the above problem using Classical Thermodynamics, not Statistical Mechanics.

<u>2.03</u> **Problem 2.1.** Suppose you flip four fair coins.

(a) Make a list of all the possible outcomes, as in Table 2.1.

(b) Make a list of all the different "macrostates" and their probabilities.

(c) Compute the multiplicity of each macrostate using the combinatorial formula 2.6, and check that these results agree with what you got by brute-force counting.

<u>2.04</u> **Problem 2.9.** Use a computer to reproduce the table and graph in Figure 2.4: two Einstein solids, each containing three harmonic oscillators, with a total of six units of energy. Then modify the table and graph to show the case where one Einstein solid contains six harmonic oscillators and the other contains four harmonic oscillators (with the total number of energy units still equal to six). Assuming that all microstates are equally likely, what is the most probable macrostate, and what is its probability? What is the least probable macrostate, and what is its probability?

2.05 **Problem 2.10.** Use a computer to produce a table and graph, like those in this section, for the case where one Einstein solid contains 200 oscillators, the other contains 100 oscillators, and there are 100 units of energy in total. What is the most probable macrostate, and what is its probability? What is the least probable macrostate, and what is its probability?

2.06 **Problem 2.13.** Fun with logarithms.

- **(a)** Simplify the expression $e^{a \ln b}$. (That is, write it in a way that doesn't involve logarithms.)
- **(b)** Assuming that $b \ll a$, prove that $\ln(a+b) \approx (\ln a) + (b/a)$. (Hint: Factor out the a from the argument of the logarithm, so that you can apply the approximation of part (d) of the previous problem.)

2.07 **Problem 2.16.** Suppose you flip 1000 coins.

- **(a)** What is the probability of getting *exactly* 500 heads and 500 tails? (Hint: First write down a formula for the total number of possible outcomes. Then, to determine the "multiplicity" of the 500-500 "macrostate," use Stirling's approximation. If you have a fancy calculator that makes Stirling's approximation unnecessary, multiply all the numbers in this problem by 10, or 100, or 1000, until Stirling's approximation becomes necessary.)
- **(b)** What is the probability of getting exactly 600 heads and 400 tails?

2.08 **Problem 2.18.** Use Stirling's approximation to show that the multiplicity of an Einstein solid, for any large values of N and q, is approximately

$$\Omega(N,q) \approx \frac{\left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N}{\sqrt{2\pi q(q+N)/N}}.$$

The square root in the denominator is merely large, and can often be neglected. However, it is needed in Problem 2.22. (Hint: First show that $\Omega = \frac{N}{q+N}\frac{(q+N)!}{q!\,N!}$. Do not neglect the $\sqrt{2\pi N}$ in Stirling's approximation.)