

Chapter 2: Thermodynamics, Temperature, and Heat

The subject of thermodynamics is as old as the hills -- well, not quite. Thermodynamics was born in the 17th century. It's central task was to study how heat might be converted into work. This is, as mentioned earlier, the task of a heat engine. Many important advances were made in the 18th century by the likes of Mayer, Joule, and Helmholtz. Among the topics where are important for us to understand in this course are heat, temperature, heat capacity, heat flow, the 2nd law of thermodynamics, and latent heat of vaporization. We begin by looking at temperature.

1. Temperature

Temperature, it turns out, is quite easy to understand and difficult to define. We are all acquainted with the idea that some objects are hot and some are cold. Temperature is the property of an object which characterizes this. The zeroeth law of thermodynamics says that two objects A and B, each in thermal equilibrium with a third object C, are in thermal equilibrium with each other. This law summarizes something you are well aware of -- think of object C as a thermometer. If two objects, again A and B, have the same temperature (measured by object C) then they are in thermal equilibrium.

There are three common scales used to measure temperature: the Fahrenheit scale (yuk!) commonly used in this country, the Celsius (or Centigrade) scale (used in the rest of the world), and the Kelvin (or absolute) temperature scale which is part of the SI. You need to be able to readily convert between these three scales. For simplicity I will use T_f to stand for the temperature of an object on the Fahrenheit scale, T_C on the Celsius scale, and T_K on the Kelvin scale. These three scales are all linear with temperature. Any line is determined by two points, in the case of temperature we use the triple point (i.e., freezing temperature) and boiling points of water. These are given in the table below. I have also given a 3rd point which is that of the *absolute zero* of temperature, the temperature at which all thermal motion stops.

scale	absolute zero	triple point	boiling point
Fahrenheit	-459.6 °F	32.0 °F	212.0 °F
Celsius	-273.1 °C	0.0 °C	100.0 °C
Kelvin	0.0 K	273.1 K	373.1 K

Notice that temperatures in Fahrenheit and Celsius are given in "degrees F" or "degrees C." In contrast, temperatures on the absolute temperature scale are given in "Kelvin" (that is, the word degree is not used). The graph below shows the three scales.

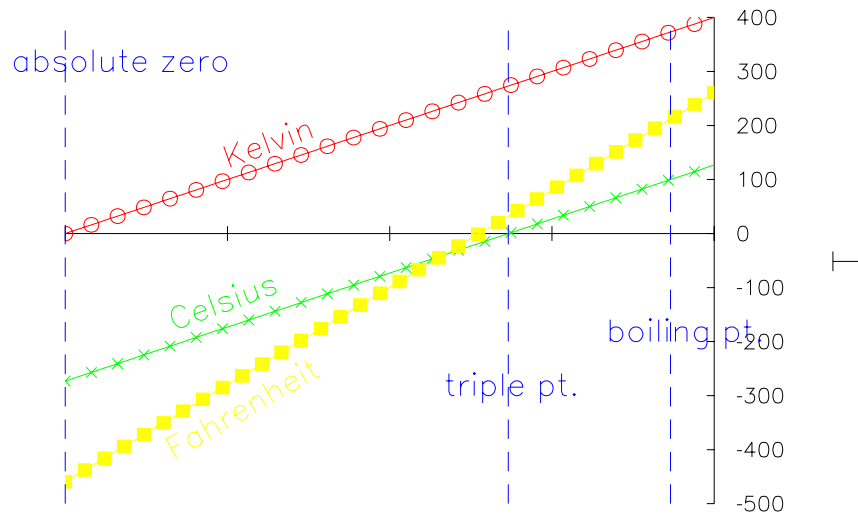


Figure 1. Plots showing the three common temperature scales.

Conversion formulas between the three temperature scales are summarized in the table below.

Scale	Fahrenheit	Celsius	Kelvin
Fahrenheit		$T_F = \frac{9^\circ F}{5^\circ C} T_C + 32^\circ F$	$T_F = \frac{9^\circ F}{5K} T_K - 459.62^\circ F$
Celsius	$T_C = \frac{5^\circ C}{9^\circ F} (T_F - 32^\circ F)$		$T_C = \frac{1^\circ C}{1K} T_K - 273.12^\circ C$
Kelvin	$T_K = \frac{5K}{9^\circ F} T_F + 255.34K$	$T_K = \frac{1K}{1^\circ C} T_C + 273.12K$	

Example 1:

Liquid nitrogen boils at a temperature of 77 K. Express this temperature in degrees Fahrenheit and Celsius.

Solution:

We are given $T_K = 77$ and must find T_F and T_C using formulas in the above table.

For Celsius this is very easy since the Kelvin and Celsius scales have the same size of one "degree", but different zeros (The lines in Figure 1 above are parallel). Thus

$$\begin{aligned}
 T_C &= T_K - 273.12 \\
 &= (77) - (273.12) \\
 &= -196.12^\circ C.
 \end{aligned}$$

For converting to Fahrenheit we have to account for different zeros and different size "degrees." These are both accomplished by the formula

$$\begin{aligned}
 T_C &= (9/5)T_K - 459.60 \\
 &= (9/5) \times (77) - (459.60) \\
 &= -321.0^\circ F.
 \end{aligned}$$

2. 1st Law of Thermodynamics

We have already discussed the fact that heat is just another form of energy. This is summarized by the *1st Law of Thermodynamics* which says that the internal energy of a system can change through either of two methods: a) by heat flowing into or out of the system, or b) by an external agent performing work on the system or the system performing work on something external. Thus, if I have a piece of aluminum at ground level, I can increase its energy by either moving it higher (performing work on it to raise its potential energy) or by heating it up.

The 1st law says that heat and work are two forms of energy. You might guess, therefore, that heat may be converted into work and work into heat -- that the two are equivalent. This is not the case. Work may be easily converted into heat (with 100% efficiency). It is not possible to convert heat into work with 100% efficiency. This limitation is summarized next.

3. Building Site Energy

The first law of thermodynamics allows us to add up all the different kinds of energy inputs to a building and convert them into one common unit. A typical building must continually consume energy to supply its needs and the needs of its occupants. The quantity and nature of the energy depends on the building, climate, and the activities it supports. Modern buildings require energy for lighting. A typical home will use natural ventilation. But commercial buildings, particularly those with many occupants, require energy for ventilation to remove CO₂ and supply fresh air. In cold weather buildings must be heated and in warm weather they are frequently air-conditioned. It is interesting to note that people and equipment generate heat. In a large office building, even in the winter, most rooms must actually be cooled, not heated, to remove the excess heat generated by the occupants and equipment.

Buildings, of course, received energy in the form of incident sunlight. Energy also enters the building through pipes and wires. Typical sources of energy are 1) electricity, 2) natural gas, 3) propane, 4) fuel oil, 5) and gasoline. Four of these represent primitive fuels – fuels that are burned to produce heat. But electricity is different – it is generated somewhere else with significant off-site inefficiencies associated with it.

In addition to the above, a building may also receive energy in processed forms from a central heating plant. Many buildings on college and university campuses are fed either steam or hot water from a central boiler for heating and cold water from a central chiller for cooling.

Energy flows into a building may be easily monitored in the units appropriate for the source. For instance, electric usage is monitored in kWh, natural gas usage is monitored in CCF (100's of cubic feet), and so on. All of these figures may be converted to a common energy unit (Btu or J) using conversion factors provided in Chapter 1.

In the case of hot or chilled water, it is a bit more complicated. In these cases you must energy both the flow of water in/out of the building as well as the supply and return temperatures. For hot water, these may be used to calculate the net energy flow into the building.

For chilled water, on the other hand, energy is actually flowing out of the building. In some sense this results in an energy “credit” to the building – at least with regard to the energy used on-site. (We will soon see that significant energy is used off-site to cool this water – more later.) We will defer the discussion of chilled water until later.

How can the on-site energy used by one building be compared with that used by another? Obviously the larger a building, the more energy it will consume. One measure of energy use of a building is its **site energy intensity**, or its **site energy**. This is the total energy used on site in a full year, divided by the gross square footage (floor area) of the building. The number is reported as so many Btu/ft²/yr or so many J/m²/yr.

Example 2:

Records show that during the 1991-92 fiscal year, the *Wright Laboratory of Physics* used 164,661 kWh of electric energy, 418 CCF of natural gas, and 1,768,000 pounds of steam. The gross square footage of Wright is 36,400 ft². Use these data to calculate its annual site energy.

Solution:

The total site energy will be the sum of the three different energies, all converted to Btu.

$$\text{Electric: } E_1 = (164,661 \text{ kWh}) \times (3.6 \times 10^6 \text{ J/kWh}) / (1054 \text{ J/Btu}) = 562,000,000 \text{ Btu}$$

$$\text{Gas: } E_2 = (418 \times 100 \text{ cf}) \times (1,031 \text{ Btu/cf}) = 562,000,000 \text{ Btu} = 43,000,000 \text{ Btu}$$

$$\text{Steam: } E_3 = (1,768,000 \text{ lbs}) \times (970 \text{ Btu/lbs}) = 1,715,000,000 \text{ Btu} = 1,715 \text{ MMBtu.}$$

The annual site energy intensity is just the sum of these divided by the area of the building, namely 63,700 Btu/ft²/yr. It is, of course, dominated by heating energy.

The above figure represents the energy per square foot consumed by the *Wright Laboratory*, on site, for a particular year. The average *site energy intensity* for Oberlin College non-residential buildings is in the neighborhood of 80,000 Btu/ft², depending on the year. For the 1995-6 academic year the number is 76,000 Btu/ft².¹

The *Energy Information Administration (EIA)* of the *U. S. Department of Energy (DOE)* compiles energy use data for thousands of non-residential buildings around the country and records them in the *Commercial Building Energy Consumption Survey (CBECS)* database for comparative purposes. This database may be parsed to obtain average site energy use for a variety of different kinds of buildings and climates. The average site energy for a 10,000-25,000 ft², educational building, in our climate, constructed since 1995 is 76,000 Btu/ft²/yr, amazingly, but only coincidentally, close to the 1995-6 Oberlin College average.

The target site energy for the *Lewis Center* was 16,500 Btu/sf/yr.² This target is about 1/5th the amount of site energy used by a comparable, conventional building. In its first two years of operation the *Lewis Center* had a measured site energy intensity of 48,000 Btu/ft²/yr, well below that of a conventional building. In its most recent 12-months of operation (Feb. 02 – Jan. 03) the building's site energy consumption has been 32,000 Btu/ft²/yr. This figure is about 42% as much site energy as an average, conventional building.

¹ John H. Scofield, "Early energy-performance for a green academic building," *ASHRAE Transaction*, **Vol. 108 Part2**, 1214-1230 (2002).

² See, for instance, Michael Reis, "The Ecology of Design," *Environmental Design & Construction*, Mar/Apr 2000.

But the energy a building consumes on site is only part of the story. It is important to account for the energy consumed off site made necessary by the delivery of the energy to the building. And, in the case of the Lewis Center, one should also account for the on-site generation of photovoltaic energy. Most buildings use electric energy that has been generated off-site at an efficiency of 30-35%. Similarly, there are distribution losses and inefficiencies in the production of steam at the central heating plant. Hence, on-site energy use, or site energy, does not well represent the total energy cost associated with building operation. Below we define a quantity called *source energy* that accounts for both on- and off-site energy consumption.

One last comment has to do with waste electric energy. All of the electric energy used in a building ultimately ends up in the form of heat. Thus, in the example above, it is not accurate to say that the energy to heat Wright all came from steam. If we lower our electric energy use, say, by installing more efficient lighting, we will then need to supply some more heat in the winter to offset the lost electric heat – more on this later.

4. 2nd Law of Thermodynamics

Much of thermodynamics deals with the fundamental problem of turning heat into work. Heat is easy to produce, mainly through combustion (wood, gasoline, coal, etc.). Work is another story. The 2nd Law of Thermodynamics says "it is not possible to change heat completely into work, with no other change taking place."³ The 2nd law has many other formulations, but this is its essence. Whenever a clever inventor claims to have invented a "perpetual motion" machine, it is eventually shown to violate the second law.

Engines are devices that convert heat into work. Basically heat flows into the engine from a "hot reservoir" and is converted into work. Of course, not all of the heat is converted into work because this would violate the second law. Instead, some of the heat gets converted into work. The remaining heat is exhausted into the cold reservoir. Thus, an engine operates between a hot and cold reservoir. A diagram is shown below.

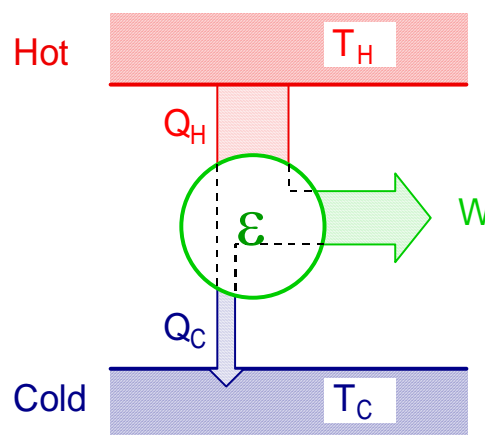


Figure 2. Diagram for an *engine* with efficiency ϵ operating between a hot and cold reservoir. An amount of heat Q_H is absorbed from the hot reservoir. The

³ D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics, Extended 4th ed.* (John Wiley & Sons, New York, 1993). p. 607.

engine converts some of the heat into work, W , and exhausts the remaining heat as Q_C into the cold reservoir. The efficiency is defined by $\varepsilon \equiv W/Q_H$. Energy conservation requires that $Q_H = W + Q_C$.

A perfectly efficient engine would take all of the heat and turn it into work -- that is, $W = Q_H$. The 2nd law says this cannot happen. There is always some waste heat.

Carnot showed that the best engine theoretically possible has an efficiency given by

$$\varepsilon = 1 - \frac{T_C}{T_H},$$

where T_C and T_H are the absolute temperatures (in Kelvin) of the cold and hot reservoirs respectively. Again, there is no limitation on converting work into heat -- this can be 100% efficient. But there are limitations on converting heat into work.

Example 3:

Aluminum metal is to be used for the block of a gasoline automobile engine. What is the maximum efficiency possible for this engine assuming that the high temperature does not melt the aluminum?

Solution:

The details of the engine are, of course, very complicated. A gasoline air mixture will burn to produce heat. This heat will ultimately heat up the aluminum metal of the engine. Aluminum melts at a temperature of 933 K. Therefore, the "hot" temperature certainly cannot be any higher than this. The waste heat will be exhausted into the atmosphere, which is roughly at a temperature of 300 K. Hence, the maximum efficiency allowed by the 2nd law of thermodynamics is

$$\begin{aligned}\varepsilon_m &= 1 - T_C/T_H \\ &= 1 - 300 / 933 \\ &= 68 \text{ \%}.\end{aligned}$$

In practice the high temperature cannot even be this high. This, combined with a variety of other factors will make the engine more like 30% efficient.

Incidentally, a refrigerator is just an engine run backwards. For a refrigerator, you supply work to the engine and this causes it to pump heat from the cold reservoir up to the hot reservoir. We will talk more about this later in connection with heat pumps.

5. Building Source Energy

Environmentalists have, for years, pointed out that many technologies carry with them hidden costs – costs to society and costs to the environment that are not necessarily reflected in the price paid for things.

When we purchase 1 kWh of electric energy, that energy is typically generated out of sight at a power plant. In Ohio, 89% of the electric energy comes from the burning of coal. The power plant efficiency ranges from 30-35%. Hence every Joule of electric energy purchased from the electric grid corresponds to about 3 Joules of heat that was used at the power plant. There is, then, significant off-site energy use associated with the on-site use of electric energy.

The true energy costs of the electric energy used by a building then is about 3 times the actual site energy to which it corresponds.

To account for this we define *source energy* to be similar to *site energy*, except that electric energy is weighted by a factor of 3 to account for the off-site energy used to produce it. An example source-energy calculation is shown below.

Example 4:

Consider again the energy used in 1991-2 by the *Wright Laboratory of Physics*. Recall that in that year it used 164,661 kWh of electric energy, 418 CCF of natural gas, and 1,768,000 pounds of steam. The gross square footage of Wright is 36,400 ft². Use these data to calculate its annual source energy.

Solution:

This calculation is identical to the site-energy example, except that we need to multiply the electric energy by 3 to account for the off-site energy used at the power plant to generate the electricity in the first place. The total site energy will be the sum of the three different energies, all converted to Btu.

Electric: $E_1 = 3 \times 562,000,000 \text{ Btu} = 1,686,000,000 \text{ Btu} = 1,686 \text{ MMBtu}$.

All other contributions (gas and steam) remain as before.

Gas: $E_2 = 43,000,000 \text{ Btu} = 43 \text{ MMBtu}$

Steam: $E_3 = 1,715,000,000 \text{ Btu} = 1,715 \text{ MMBtu}$.

The total source energy then is 3,444 MMBtu.

The annual source energy intensity is just the sum of these divided by the area of the building, namely 94,600 Btu/ft²/yr. This is 50% higher than the site energy, and is a better measure of the true energy costs of a building.

You are right to ask if there aren't some off-site energy costs associated with the delivery of steam heat and natural gas to the building as well. The answer is yes, but they are a good deal smaller and harder to quantify. For instance, the campus steam system operates at about 75-80% efficiency. You could say, then, the source energy associated with steam heat should be 25-30% larger than the site energy. But this is not nearly as important as the 200% correction for electric energy purchased from the grid.

The CBECS database mentioned above also compiles source energy for thousands of commercial buildings, so defined. The average source energy for a 10,000-25,000 ft², educational building, in our climate, constructed since 1995 is roughly 130,000 Btu/ft²/yr.

6. Heat Capacity

If you add energy to almost any system its temperature will increase. The *heat capacity* of a system is the ratio of the heat added ΔQ to the temperature increase ΔT , namely

$$C \equiv \frac{\Delta Q}{\Delta T}.$$

The units of heat capacity are energy per unit temperature. In the MKS system this is Joules/Kelvin. The heat capacity of 1 cc of liquid water at STP is 1 cal/°C = 4.18 J/K. This

means you have to add 4.18 Joules of energy to one ml of water to raise its temperature from 300 K (room temperature) to 301 K. If instead you have 1 liter of water, it will take 1000 times this amount of energy to accomplish the same 1K increase in temperature. The heat capacity is proportional to the size of the system. Because of this it is convenient to define the *specific heat*, c_s . Like the density (mass per unit volume) the specific heat is an "intensive" property of a substance that does not change with the quantity of the substance. The specific heat of water is 1 cal/g/°C. If a substance has a specific heat c_s and a mass M , then its heat capacity is given by

$$C \equiv c_s M .$$

The specific heats for several common substances are listed in the table below.

substance	c_s (J/kg/°C)	δ (kg/m ³)
aluminum	898	2700
steel	447	7800
copper	385	8900
iron	443	7800
nitrogen N ₂ (g)	1040	1.25
brick		1400-2200
wood (pine)	1800	350-600
wood (oak)		600-900
Styrofoam		100
granite		2700
glass	840	
water	4169	1000
ice (-10 to 0°C)	2089	917
steam	2000	

Table I. Specific heats c_s (at constant pressure) and densities for several common substances. Unless otherwise noted, values are given at STP.

The heat capacity of a gas depends upon the measurement conditions. The two conditions that are common are *constant volume* or *constant pressure*, and heat capacities measured under these conditions are denoted C_V and C_P respectively. If you add some heat to a gas that has fixed volume (a rigid container) then the gas cannot expand. This keeps the gas from doing any work so that all of the heat added goes into raising its temperature. On the other hand, you might hold the gas in a "balloon-like" container, which is allowed to expand against atmospheric pressure. As you add heat, the gas expands, causing some of the energy you add to be "spent" in work against the atmosphere, rather than in raising the gas temperature. Thus, the apparent heat capacity measured under constant pressure will be greater than that measured for constant volume. This distinction is of little importance for liquids and solids as they won't expand much, but it is very important for gases.

Example 5:

A homeowner wishes to heat up the water in his swimming pool from 65 °F to 80 °F. If the swimming pool is 15 feet wide by 8 feet deep by 20 feet long, how much energy is required?

Solution:

The increase in temperature $\Delta T = 15\text{ }^\circ\text{F}$ or $15/1.8\text{ K} = 8.3\text{ K}$. The volume of water is

$$\begin{aligned} V &= (15\text{ ft.}) \times (8\text{ ft.}) \times (20\text{ ft.}) \\ &= 2,400\text{ cu. ft.} \times (7.48\text{ gal/cu. ft.}) \\ &= 17,950\text{ gal} \times (3.78\text{ lit/gal}) \\ &= 67,950\text{ lit.} \end{aligned}$$

The δ of water is $1\text{ g/cc} = 10^3\text{ g/lit}$, so, the mass of the water is

$$M = \delta V = 6.79 \times 10^7\text{ g.}$$

The heat capacity is then

$$\begin{aligned} C &= MC_S = (6.79 \times 10^7\text{ g}) \times (4.18\text{ J/g/}^\circ\text{C}) \\ &= 2.84 \times 10^8\text{ J/K} \quad (\text{since } 1\text{K} = 1^\circ\text{C}). \end{aligned}$$

The required heat then is $\Delta Q = C \Delta T$, or

$$\begin{aligned} \Delta Q &= (2.84 \times 10^8\text{ J/K}) \times (8.3\text{ K}) \\ &= 2.37 \times 10^9\text{ J.} \end{aligned}$$

Recall that $1\text{ kW}\cdot\text{hr} = 3.6 \times 10^6\text{ J}$. If the pool were heated up using electric power at a cost of \$0.06 per kW·hr, the cost would be nearly \$40.

7. Latent Heat

As mentioned above, the temperature of a body generally increases as you add heat. The graph below shows in more detail how the heat capacity of a system may be obtained from a graph of its temperature versus heat added. The heat capacity is the inverse of the slope of the graph.

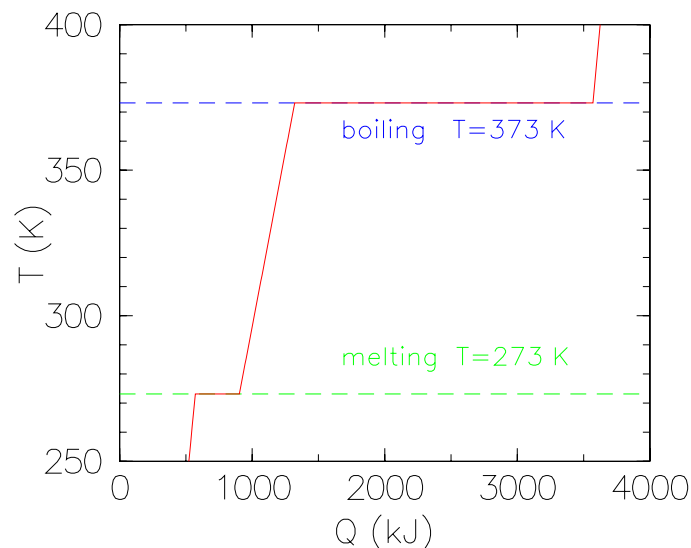


Figure 3. Graph of the temperature of a water sample versus heat added.

Usually the temperature of a system goes up as you add heat. An important exception occurs, however, when the substance undergoes a phase transition, as is the case when a substance melts or vaporizes. Consider, for instance, the heating of solid water (ice). As heat is added to ice, its temperature goes up. When the ice temperature finally reaches 273.12 K the ice begins to melt. This takes energy -- that is, when the ice reaches a temperature of 0 °C it wants to melt, but it needs additional energy to convert the solid to liquid. Thus, as you add heat some of the solid melts. Both the remaining ice and the liquid remain at a temperature of 0 °C. As more heat is added, the temperature remains the same, but more of the remaining ice melts, and so on. Eventually, when all of the ice has melted, any further heat you add will begin to raise the temperature, now of the liquid water. This transition corresponds to the "flat" spot on the above graph near $T = 273.12\text{K}$. A similar thing occurs at the temperature where the water vaporizes. Curiously the slope of the graph is zero at these places. Putting in a positive amount of heat ΔQ results in zero increase in temperature ΔT -- hence the ratio $\Delta Q/\Delta T$ is infinite. Physicists think this is really neat, but it isn't of much consequence for us.

The *latent heat of fusion* is the amount of energy it takes to convert the solid to liquid. This will scale with the amount of substance you have. Hence it is more convenient to express the latent heat as a quantity of energy per mole or per unit mass. The latent heat of fusion for water is **79.7 cal/g**.

A similar effect occurs at the boiling point. The amount of heat that must be added to a liquid to convert it into vapor at its boiling point is called the *latent heat of vaporization*, again, usually expressed in an amount of energy per mole or unit mass. The latent heat of vaporization for water is **539 cal/g**.

Vaporization is very important for cooling. In hot weather our bodies produce sweat in order to get rid of heat. Sweat (water) evaporates from the surface of our bodies. This evaporation requires energy, heat that is absorbed from the body, leaving the body cooler!⁴ A similar effect is often used to cool buildings – more on this later.

8. Fourier's Heat Law

We all know that heat will flow from a hot body to a cold body. Consider the following situation. A plate of glass having thickness d is "sandwiched" between two blocks of copper. (I have chosen copper because it conducts heat so well it is easy to imagine that each of the copper plates is at a uniform temperature.) One of the copper plates has a temperature $T_2 = 50\text{ °C}$ while the other has a temperature $T_1 = 25\text{ °C}$. All of the plates have the same area, $A = LW = 0.5\text{ m}^2$. At what rate will heat flow from the hotter plate to the cooler plate through the glass? The situation is diagrammed below.

⁴ Incidentally, dogs do not sweat. They cool themselves by panting, instead.

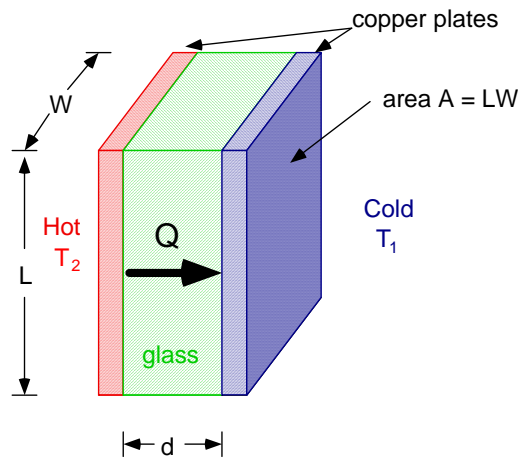


Figure 4. Heat flows from a hot copper plate at temperature T_2 through a piece of glass to another copper plate at temperature T_1 .

The flow of heat in this situation is described by Fourier's heat law. Before writing down the law keep in mind the following intuitive ideas.

- If the two copper plates are at the same temperature ($T_1 = T_2$) then no heat will flow.
- The thicker the glass, the less heat that will flow.
- If the glass were replaced with something that conducted heat better, say, a metal sheet, then more heat will flow. In other words, the amount of heat that flows ought to depend on some property of the glass which indicates how well it conducts heat, independent of its size.
- The amount of heat that flows will increase with the area A .

With these ideas in mind, here is the law that describes what happens:

$$P = \frac{\kappa A}{d}(T_2 - T_1),$$

where P is the power that flows (energy per unit time) and κ is the *thermal conductivity* of the glass. A table of thermal conductivities for various materials is given below. As with all problems we first attack the problem in MKS units. Later, when we make practical calculations we may be forced to use other units. The MKS unit for thermal conductivity is W/K/m. Unfortunately, as indicated in the table below, a variety of different units are commonly employed.

substance	density δ (g/cc)	specific heat c_s (cal/g/°C)	thermal conductivity κ (cal/s/cm/°C)
copper	8.94	0.0914	0.93
aluminum	2.70	0.206	0.48
mild steel	7.85	0.118	0.11
air	0.00129	0.240	0.000058
granite	2.6	0.21	0.006
ice	0.92	0.502	0.0053
crown glass	2.4	0.20	0.0028
concrete (1:2:4)	2.3	0.23	0.0022
wood (spruce)	0.41	0.30	0.00055
cross grain	0.41	0.30	0.00030
water	1.0	1.0	0.00144

Table II. List of thermal properties of some building materials. Numbers are given in cgs units (cal, cm, s, etc.). Data taken from Carslaw & Jaeger, *Conduction of Heat in Solids*, Appendix 6.

Example 6:

The north wall of an Oberlin home is made from solid wood. The wall is 4 inches thick and has an area of 20 ft. x 15 ft. Calculate the rate of heat loss through this wall on a winter day when the interior temperature is 70 °F and the outside temperature is 20 °F.

Solution:

We will treat this as a simple heat conduction problem. First convert all of the relevant numbers to MKS units.

$$\kappa = (0.00055 \text{ cal/s/cm/}^\circ\text{C}) \times (4.18 \text{ J/cal}) \times (100 \text{ cm/m})$$

$$= 0.230 \text{ W/m/K.}$$

$$A = (20 \text{ ft.} \times 15 \text{ ft.}) \times (1 \text{ m}/3.28 \text{ ft.})^2$$

$$= 27.9 \text{ m}^2.$$

$$d = 4 \text{ in.} = 0.102 \text{ m.}$$

$$\Delta T = (70 - 20) \text{ }^\circ\text{F} \times 5/9 \text{ (}^\circ\text{C/}^\circ\text{F)}$$

$$= 27.8 \text{ K.}$$

$$P = \kappa \Delta T A / d$$

$$= (0.23) (27.8) (27.9) / 0.102$$

$$= 1.75 \text{ kW.}$$

In a few weeks we will look again at heat conduction, particularly as it applies to heat loss through building walls and windows. Many building materials are standardized -- that is, come in common thicknesses and use common materials. Accordingly it is convenient to combine the thermal conductivity and thickness into one ratio. This ratio is called the "thermal resistance" or "R-value," namely

$$R \equiv \frac{d}{\kappa}$$

R-values for common building materials may be found in units of $\text{ft}^2\text{-}^\circ\text{F}\text{-hr/Btu}$.

One final comment has to do with the assumptions that go into Fourier's heat law, and whether they may be simply applied to common situations. In the above example we calculated the heat flow through a 4-in-thick wood wall. The calculated heat flow is about what would be observed, so Fourier's heat law is a good model for the situation. Consider, a second example below, heat flow through a 1/8-in-thick glass window.

Example 7:

A window is 3 ft. wide and 9 ft. high, and is made from crown glass having a thickness of 1/8 in. Calculate the rate of heat loss through this window on a winter night when the interior temperature is 70 °F and the outside temperature is 20 °F.

Solution:

We solve this as we did the previous example. First, convert to MKS units:

$$\begin{aligned}\kappa &= (0.0028 \text{ cal/s/cm/}^\circ\text{C}) \times (4.18 \text{ J/cal}) \times (100 \text{ cm/m}) \\ &= 1.17 \text{ W/m/K}\end{aligned}$$

$$\begin{aligned}A &= (3 \text{ ft.} \times 9 \text{ ft.}) \times (1 \text{ m}/3.28 \text{ ft.})^2 \\ &= 8.23 \text{ m}^2\end{aligned}$$

$$d = 0.125 \text{ in.} = 3.18 \times 10^{-3} \text{ m}$$

$$\begin{aligned}\Delta T &= (70 - 20) \text{ }^\circ\text{F} \times 5/9 \text{ (}^\circ\text{C/}^\circ\text{F)} \\ &= 27.8 \text{ K}\end{aligned}$$

Finally, combine with Fourier's heat law:

$$\begin{aligned}P &= \kappa \Delta T A / d \\ &= (1.17) (27.8) (8.23) / (3.18 \times 10^{-3}) \\ &= 84 \text{ kW}\end{aligned}$$

This is a huge heat loss through a window. The number implies that it would take 84 toasters sitting in front of the window to offset the heat loss. This is not believable! In fact, the amount of heat loss through such a window would typically be more like 400 W, 1/200 of the above value.

The problem with the above example is that the assumptions of Fourier's heat law are not satisfied by a real window. If the inside and outside surfaces of the window were maintained at the two temperatures, the calculation would be correct. But with such a huge amount of heat loss through the window, the inside surface will not stay at 72 °F -- it will get much colder. (Many of you have probably seen ice form on the inside of a single pain bathroom window after taking a shower in the winter!) Similarly, the outside surface will not stay at 20 °F -- it will get slightly warmer. In "steady-state" the actual temperature difference across the glass itself will be much less than the 52 °F difference between the interior and exterior of the building. Essentially a cold layer of air will form on the inside of the window, adding to the insulation and cutting down on the heat loss.

Thus, calculated heat losses using just Fourier's heat law tend to be quite inaccurate for poorly insulated walls. They are better for well-insulated walls. In a later we will talk more about effective R-values that are measured, rather than calculated, so may be used to calculate heat loss through actual walls and windows.