Chapter 1: Energy, Work, and Power

Energy is a very important concept both in physics and in our world at large. Energy takes various forms. A massive truck traveling along the highway at a high speed has much kinetic energy; a water reservoir just above a dam contains significant gravitational potential energy; a tank of gasoline contains significant chemical energy; radio waves emitted from a broadcast antenna contain energy stored in the electric and magnetic fields.

Energy can take on many forms but it is never created or destroyed. This is one of the fundamental laws of physics. We say that energy is *conserved*. Various forms of energy include heat, light, radio waves, translational kinetic energy, rotational kinetic energy, gravitational potential energy, chemical energy, and electric potential energy. Even mass is a form of energy as Einstein showed in his famous formula, $E = mc^2$.

In this short tutorial I hope to present the basic energy concepts that you will need to consider practical problems involving building energy. This tutorial cannot hope to substitute for a physics course.

1. System of Units and Conversions

In physics, work has a precise definition involving a force and distance. Scientists world wide have adapted a system of measurements called System International (SI). In this system length is measured in **meters** (m), mass in **kilograms** (kg), and time in **seconds** (s). The subset of the SI units which are commonly used in physics courses is called the *MKS system* (for meter-kilogram-second). In SI units temperature is measured in Kelvin (K). A less well-known quantity is charge. The SI unit for quantity of charge is the **Coulomb** (C). One coulomb is equal to the total charge of 6.24 x 10^{18} protons.

Many other quantities are derived from these basic units and are given their own names. Below is a summary of the main quantities of interest in this course.

Quantity	MKS Unit	Definition
force	Newton (N)	1 N = kg m/s^2
energy	Joule (J)	1 J = N m
power	Watt (W)	1 W = J/s
electric potential	Volt (V)	1 V = J/C
current	Ampere (A)	1 A = 1 C/s
electric resistance	Ohm (Ω)	1 Ω = V/A

Table 1. Units in the MKS system along with basic conversion factors.

Unfortunately, particularly in the United States, a variety of measurements are made using other systems of units. This is especially true in the construction industry. One such system is the *British system* of units (long abandoned in Great Britain, of course) which measures length in feet, mass in slugs, and time in seconds. You will need to be able to readily convert between the various systems of units. Here is a list of various quantities and the common units each quantity might be measured in. You must be able to convert between any of these different measures.[1]

Quantity	MKS unit	Other common units
length	meter	centimeter (cm), foot (ft), inch (in), yard (yd), mile (mi)
time	second	minute, hour, day, year
volume	cubic meters	cc, liters, gallons, quart, pint, cubic feet, cubic inches
weight	kilogram	pounds (lbs), ounces (oz.), grams (g), ton, metric ton
energy	Joule	calorie, Calorie, British thermal unit (btu), kilowatt-hour, erg
power	Watt	btu per hour, horse-power (hp)
temperature	Kelvin	degrees Celsius (°C), degrees Fahrenheit (°F)

Table 2. List of common physical quantities and the various units used to measure them

Relevant conversion factors are given below.

1 ft = 12 in	1 in = 2.54 cm	3 ft = 1 yd	5280 ft = 1 mi	1 m = 3.28 ft
60 sec = 1 min	60 mins = 1 hr	24 hr = 1 day	365 day = 1 yr	
1 $m^3 = 10^6$ cm^3	1 lit = 10^3 cm^3	4 qt = 1 gal	2 pt = 1 qt	1 in^3 = 16.4 cm^3
	1 ft^3 = 1728 in^3	1 gal = 3.785 lit	1 m^3 = 35.31 ft^3	1 ft^3 = 7.48 gal
1 lb = 16 oz	1 lb = 454 g	1 kg = 1000 g	1 ton = 2000 lbs	1 me. ton = 1000 kg
1 cal = 4.18 J	1 Cal = 1000 cal	1 btu = 1054 J	1 kw-hr = 3.6 x 10^6 J	1 J = 10^7 erg
1 hp = 745.7 W	1 W = 3.414 btu/hr			

Table 3. Common conversion factors for length, time, volume, energy, power, and weight.

In addition to the above you should be familiar with the common prefixes used in the metric system. These are:

prefix	value	prefix	value
pico	10^{-12}	kilo	10^3
nano	10^{-9}	mega	10^6
micro	10^{-6}	giga	10^9
milli	10^{-3}	terra	10^{12}
centi	10^{-2}		

Table 4. Common metric prefixes and their definitions.

1 Weight is technically a force and, in the MKS system, is measured in Newtons. When one is asked to find the weight in kilograms you are actually being asked to find the mass whose weight is so many pounts.

Example 1:
The recommended air change for a commercial building is a rate of 20 cfm (cubic feet per minute) per occupant. Calculate the recommended air exchange rate Q in liters per second assuming the building has 100 occupants.

Solution:
Q = (20 ft^3/min/person) x (100 per) x (1 min/60 s) x (7.48 gal/ft^3) x (3.785 lit/gal)
= 944 lit/s
You may recall that air is mainly made up of nitrogen gas (N_2), and that at STP, 1 mole of a gas occupies 22.4 liters. Thus, this corresponds to 42.1 moles of nitrogen per second.

2. Kinetic energy

2.1 Translational Kinetic Energy

One of the most important forms of energy is kinetic energy. This is energy associated with the speed of an object. If a body has mass m and speed v then its (translational) kinetic energy is given by

$$K_t = \tfrac{1}{2} m v^2 .$$

Note that while the velocity of an object may be positive or negative (depending on direction), the kinetic energy is always positive. It is impossible for an object to have negative kinetic energy.[2]

Example 2:
Calculate the kinetic energy of a 2-ton truck traveling at a speed of 55 mph.

Solution:
First convert mass and speed to MKS units:
m = (2 ton) x (2000 lbs/ton) x (0.454 kg/lbs)
= 1820 kg
v = (55 mi/hr) x (5280 ft/mi) x (1 m/3.28 ft) x (1 hr/60 min) x (1 min/60 s)
= 24.6 m/s
K_t = (1/2) x (1820 kg) x $(24.6 \text{ m/s})^2$
= 5.50 x 10^5 J
This is roughly the energy used in 2 hours by a 60 watt light bulb

2.2 Rotational Kinetic Energy

A rigid body (i. e., an object, as opposed to a point mass) may also be undergoing rotation. This rotation also represents kinetic energy, but not given by the above formula which applies

[2] Here we are discussing classical mechanics. In the theory of quantum mechanics, relevant to subatomic particles, a particle may perform quantum mechanical tunneling through a region in which it has negative kinetic energy -- a strange situation confirmed by experiments.

only to translation. A rotating flywheel, for example, can store energy in the form of its rotational kinetic energy! The rotational kinetic energy of a body having moment of inertia I and angular velocity ω is given by

$$K_r = \tfrac{1}{2} I\omega^2 .$$

The symbol ω is the greek letter "omega." (Physicists love to use greek letters – I guess it makes us feel smart.) In the MKS system angular velocity is measured in radians per second (1 revolution/s = 6.14 rad/s) and moment of inertia has units of kg m^2. Moments of inertia are rather complicated to calculate and depend upon the axis of rotation. The most important case is for a solid cylinder of radius R, mass M, spinning around its symmetry axis. For this situation its moment of inertia is $I = \tfrac{1}{2} MR^2$. The reader is referred to a standard physics textbook for more on moments of inertia and rotational kinetic energy.

Example 3:
When standard brakes are applied to slow the speed of an automobile its kinetic energy is converted into heat, and thus wasted. Automotive engineers are developing "braking systems" which convert the translational kinetic energy of the vehicle into rotational kinetic energy of a massive disk located in the trunk. Calculate the rotational kinetic energy of a solid disk of mass 100 kg, radius 20 cm, rotating at a rate of 5000 rpm.

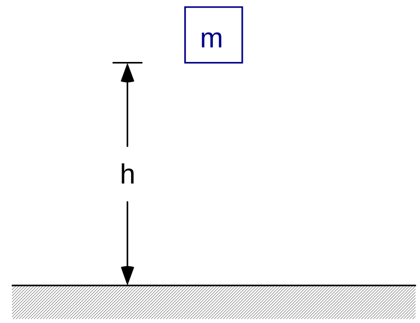
Solution:
First, calculate the moment of inertia of the disk
I = $(1/2)MR^2$
= $(1/2) \times (100 \text{ kg}) \times (0.2 \text{ m})^2$
= 2 kg m^2
Next, calculate the angular velocity
ω = (5000 rev./min) x (2π rad/rev) x (1 min/60s)
= 524 rad/s

Finally, calculate the rotational kinetic energy
K_r = $(1/2)I\omega^2$
= $(1/2) \times (2 \text{ kg m}^2) \times (524 \text{ rad/s})^2$
= 2.74×10^5 J
This is roughly the energy used in 1 hour by a 60 Watt light bulb.

Rotation gives rise to rotational kinetic energy. So far I have been discussing what is called translational kinetic energy, the energy of an object which is associated with its linear velocity. A body may not have a net velocity, but it may be rotating about some axis. This rotational motion gives rise to what is called rotational kinetic energy. The rotational kinetic energy is related to the rotational velocity

3. Gravitational potential energy

There are many forms of potential energy. One common form is the potential energy of a mass m that is a height h above the ground. The gravitational potential energy E_g is given by



$$E_g = mgh$$

where g = 9.8 m/s^2 is the acceleration due to gravity.

This represents potential energy. If the mass is released and allowed to fall under the influence of gravity it will accelerate towards the ground, ultimately hitting the ground with a certain amount of kinetic energy. The kinetic energy with which it hits the earth is exactly equal to the potential energy it had before being released (neglecting air resistance).

Example 4:
A certain water tower is 100 feet high and holds 100,000 gallons of water. What is the gravitational potential energy stored in this tank?

Solution:
First find the mass of the water. Recall the density (mass per unit volume) is 1 g/cm^3.
m = (10^5 gal) x (3.785 lit/gal) x (1000 cc/lit) x (1 g/cc) x (1kg/1000 g)
= 3.79 x 10^5 kg
Next, calculate the height in meters:
h = (100 ft) x (1 m/3.28 ft) = 30.5 m

E_g = mgh
= (3.79 x 10^5 kg) x (9.8 m/s^2) x (30.5 m)
= 1.13 x 10^8 J
This is roughly 200 times the kinetic energy of the 2-ton pickup in the an earlier example.

4. Work and Power

4.1 Work

In every day usage the word "work" has several meanings. You go to work in order to draw a pay check. Work is something that is difficult to do -- it makes you sweat. Work is an activity you do not enjoy, in contrast to play.

In physics work has a precise definition. When a force $\vec{F}$ acts on an object while it undergoes a small displacement $\Delta\vec{s}$, we say that the work done by the force on the object is given by the "dot product" between the force and displacement vectors, namely,

$$W = \vec{F} \cdot \Delta\vec{s}$$

A vector has both magnitude and direction. If the displacement and force vectors are both in the same direction then $W = F\Delta s$. If θ is the angle between $\vec{F}$ and $\Delta\vec{s}$ then the work given by

$$W = F\Delta s\cos\theta .$$

If the force is in a direction perpendicular to the displacement then no work is done. If the force is actually in the opposite direction as the displacement then the force performs negative work (since cos(180°) = –1). Maximum work is performed if the force and displacement are in the same direction.

Example 5:
A UPS delivery man pushes on a box with a force of 100 N while sliding it along the floor a distance of 5 meters at constant speed. How much work is performed by the UPS man in moving the box?

Solution:
In this case the force and displacement are in the same direction so W = FΔs.
W = (100 N) x (5 m)
= 500 J.

4.2 Work Performed Compressing a Gas

Recall from your chemistry class that an "ideal gas" is one which obeys the ideal gas law, namely

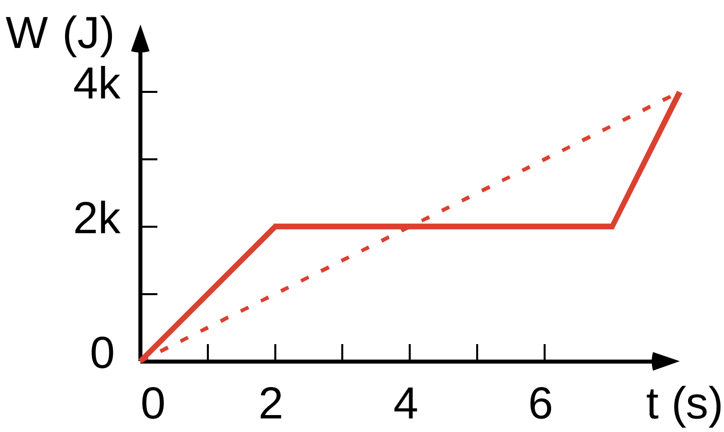
$$pV = nRT ,$$

where p is the gas pressure, V is its volume, T is its temperature, n is the number of moles, and R is the ideal gas constant, R = 8.31 J/K. Pressure p is force per unit area. When an external force compresses a gas at pressure p by a volume ΔV it performs an amount of work on the gas given by

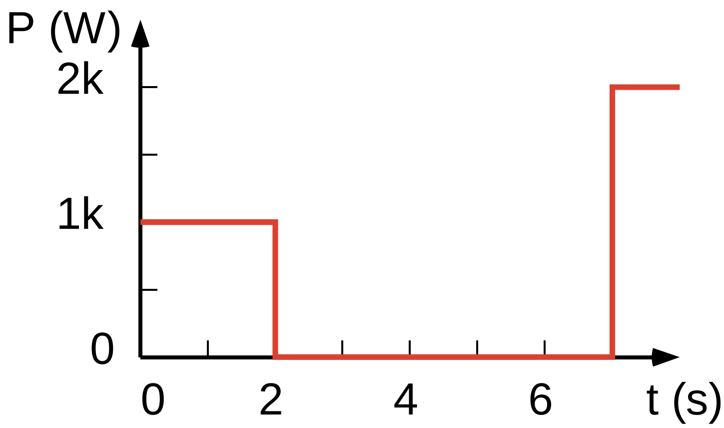
$$\Delta W = p\Delta V$$

4.3 Power

Power is the rate at which work is performed. For those who know calculus, the *instantaneous power* is the time derivative of the work, that is, $P = \frac{dW}{dt}$. If we make a graph of the amount of work performed as a function of time, the instantaneous power (rate of doing work) is the slope of the graph at any point in time. This is illustrated in the graph below.



Initially, no work has been performed. In the initial 2 seconds, the force performs 2000 J (2 kJ) of work. The power during this time is (2 kJ)/(2 s) = 1k J/s or 1 kW (kilowatts). During the next 5 seconds, no additional work is performed, and the power during this time is zero. After 7 seconds, there is a burst of energy. In one second an additional 2 kJ of work is performed. The rate of work during this time is 2 kJ/1s = 2 kW. The average power performed in the 8 seconds shown is just (4 kJ)/(8 s) = 500 W. The dashed line angled up to the right shows a force performing work at the constant rate of 500 W. Below is a graph of the power versus time for this same situation.



If an amount of work ###W is performed in a time Δt the average power is given by

$$P_{avg} = \frac{\Delta W}{\Delta t}.$$

If work is being performed at a constant rate than the average power is the same as the power.

<u>Example 6:</u>
Calculate the amount of energy used by a 100 W light bulb over a 24-hour period.

Solution:
Here we are given Δt and P_{avg} and use the above formula to calculate ΔW.

$\Delta W = P_{avg}\,\Delta t$
$= (100\ J/s) \times (24\ hr) \times (3600\ s/hr)$
$= 8.64 \times 10^6\ J$

5. Heat

Heat has been defined as energy in motion. Heat will "flow" from a hot object to a cold object when they are brought into thermal contact. For years scientists did not realize the equivalence of heat and energy. Historically heat was measured in calories. Experiments by Joule showed that heat was another form of energy. He deduced the conversion that 1 cal = 4.18 Joules. A calorie is such a small unit of energy for chemical reactions that chemists tend to work in kilocalories (1000 calories) which, unfortunately are also referred to as 1 Calorie (with a "big C"). It is the "big calorie" that is used to discuss the energy content of various foods. The standard daily food intake of an adult male is something like 2000 Cal = 2×10^6 cal = 8.36×10^6 J.

The work of frictional forces result in heat. A car traveling at a given speed has kinetic energy. If the brakes are applied the force of friction between the brake pads and the rotors performs work When the car come to a stop it is because the work of friction has added a negative amount of energy to the car. Ultimately the kinetic energy of the car is turned into heat by the frictional forces.

6. Chemical Energy

Recall from your chemistry class that an exothermic reaction occurs when two or more reactancts combine to form a product which is more stable. In such a reaction, the products have lower energy than the reactants. The extra energy is released in the form of heat. An endothermic reaction is one in which the products actually have higher energy than the reactants. In this case heat must be supplied to enable the reaction to go forward.

One way to think about such reactions is that an endothermic reaction is one in which you are storing energy chemically. In an exothermic reaction you get the energy back in the form of evolved heat.

Example 7:

The thermite reaction is highly exothermic:

$$2Al(s)+Fe_2O_3(s) \rightarrow 2Fe(s)+Al_2O_3(s), \quad \Delta H = -202.6 \text{ kcal/mole}$$

Calculate the energy released when 36 g of Al reacts with excess $Fe_2O_3(s)$.

Solution:

Refer to a periodic table to see that the atomic mass for aluminum is 27.0 g/mole. The above chemical equation tells us that two moles of aluminum combine with excess iron oxide to release 202.6 kcal of heat. In our case we have n = 36.0g / 27.0g = 1.33 moles. Thus, the amount of heat released will be

$$E = (1.33/2.00)\ 202.6 \text{ kcal}$$
$$= 135 \text{ kcal}$$
$$= 5.65 \times 10^5 \text{ J}$$

There are a number of common fuels which may be oxidized (i. e., burned in air) to release heat energy. With the exception of the last row, the table below summarizes the energy content in a variety of fuels.

Fuel	Qty	Heat (Btu)	Reference
gasoline	1 gallon	125,000	[4]
natural gas	1 cu. ft.	1,031	[4]
oil	1 pound	19,000	[3]
propane	1 gallon	91,600	[4]
dry wood	1 pound	6,900	[3]
hydrogen	1 pound	61,000	[3]
coal	1 pound	12,000	[4]
steam	1 pound	970	

Table 5. Energy content of various fuels.

The last row in the above table needs some extra discussion. Water is, of course, not a fuel, but steam does provide an important medium for distributing thermal energy. Energy must be added to water to convert it into steam. That steam may then be distributed through pipes to provide heat to remote locations. When heat is removed from the steam it condenses back into water. The (hot) water is subsequently returned to the boiler where heat is added again to make steam. Ideally all of this takes place at a constant temperature, namely, the boiling point of water.

The physics of this process will be discussed later but before we get there, it is useful to know how much heat energy is released by the condensation of 1 pound of steam. This figure is 970 Btu. To within 3% you can estimate the heat energy in steam to be 1000 Btu/lb. This is a useful number when determining the heat energy supplied to a building by the College's central heating plant.

How do we measure the weight of the steam? We usually don't. Instead we measure the volume, or weight of the condensed water that returns to the boiler.

7. Electromagnetic Waves

When charged particles (electrons, protons, etc.) are accelerated they emit electromagnetic waves. Electromagnetic waves consist of mutually perpendicular, oscillating electric and magnetic fields. In free space (i. e., empty space), EM waves travel with a constant speed c = 3.00×10^8 m/s called the speed of light since light is the most common type of electromagnetic wave. Like all periodic waves, electromagnetic waves are characterized by a frequency f and a wavelength λ. The two are always related by

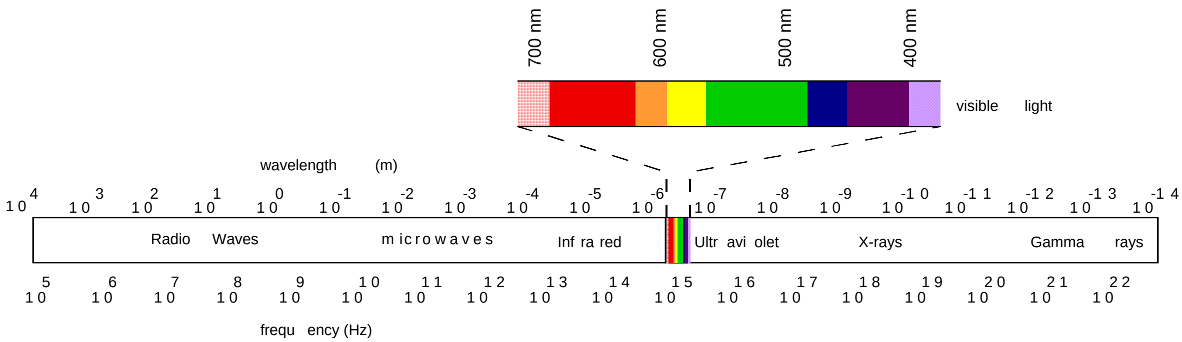
$$\lambda f = c\,.$$

In the MKS system frequency is measured in cycles per second or Hertz (Hz) and wavelength is measured in meters. Electromagnetic waves can have any wavelength as short as zero and as long as infinity. Our eyes are sensitive to EM waves having wavelengths between 400 nm

[3] *Harnessing Hydrogen: The Key to Sustainable Transportation* by James S. Cannon. <http://www.ttcorp.com/nha/harness1.htm>.

[4] UGI Utilities, Inc. web page <http://www.ugi.com/gas/factors.htm>.

(violet) and 700 nm (red). This is the spectrum of visible light. Wavelengths slightly shorter than these correspond to ultraviolet light while slightly longer wavelengths correspond to infra-red radiation. As the wavelength becomes even shorter the radiation becomes X-rays, and shorter yet, gamma rays. Longer wavelengths become microwaves, and longer yet turn into radio waves. The Electromagnetic spectrum is illustrated below.



As mentioned above, the speed of light through free space is c. Light travels through other media as well, but generally with a slower speed. A given media is characterized by an index of refraction, a dimensionless number greater or equal to unity. The speed of light v through a media having index of refraction n is

$$c = \frac{v}{n}.$$

Window glass has an index of refraction close to 1.5. Air has an index of refraction that is very close to 1.00 (i. e., the same as empty space). Water has an index of refraction of 1.33. If an electromagnetic wave of frequency f travels from one media into another (for example, from air to glass) its frequency remains the same but its wavelength changes due to its change in speed.

Example 8:

Calculate the wavelength in air of radio waves broadcast by WOBC at a frequency of 93 MHz. What is the wavelength in water?

Solution:

In air, n = 1.00 so v = c.

λ_{air} = c/f

= $(3 \times 10^8$ m/s) / $(9.3 \times 10^7$ Hz)

= 3.23 m

In water n = 1.33 so

λ_{water} = (3.23 m) / (1.33)

= 2.43 m

Light and other electromagnetic waves carry energy. The light emitted from a light bulb carriers energy away from it at the same rate as it is being produced. Of common interest are plane waves, circular waves, and spherical waves. A pebble dropped into a quiet pool of water will generate a circular wave -- wave fronts travel away from the initial disturbance on the surface of the water in the form of concentric circles. In the absence of losses, the total energy contained in a wave front remains constant. But as the wave moves farther from the source the wave front spreads out over a greater and greater region of space. Thus, the wave gets less intense, but the intensity times the area of the wave front remains a constant.

A light bulb emits waves which spread spherically. The intensity of the wave is the power per unit area of the wave front. If the total energy in a spherical wave front is, say, 100 J, then, when the wave is a distance of 10 meters away from the source, all that energy will be uniformly spread out over the area of a sphere of radius 10 m. Thus, the intensity as a function of distance r from the source is given by

$$I = \frac{P}{A} = \frac{P}{4\pi r^2},$$

where P is the source power and A is the area of the wave front. When the wave has gotten 20 meters from the source its intensity has decreased further by a factor of 4.

<u>Example 9:</u>
The sun emits radiation with a total power of 3.91×10^{26} W. What is the intensity of the radiation when it reaches the earth?

Solution:
Light from the sun spreads to spherically. The earth is a distance of 1.5×10^{11} m from the sun, so at the earth, the total power emitted by the sun has spread out over a sphere of this radius. Thus

$I = P/A = P / (4\pi r^2)$
$= (3.91 \times 10^{26}\ \text{W}) / (4) / (3.14) /(1.5 \times 10^{11}\ \text{m})^2$
$= 1380\ \text{W/m}^2$

This is roughly the intensity of sunlight at the equator at noon.

8. Einstein's Mass/Energy Relation

For years scientists thought that mass remained a constant. In the early 1900's Albert Einstein, as a consequence of his theory of special relativity, showed that mass was simply another form of energy! The amount of energy associated with a mass m is given by

$$E = mc^2$$

where $c = 3.00 \times 10^8$ m/s is the speed of light. The amount of energy associated with even the smallest amount of mass is simply incredible. It takes unusual circumstances to cause the mass to be converted into another form of energy -- this occurs in a nuclear power plant, nuclear weapon, or in a star.

<u>Example 10:</u>
Calculate the amount of energy contained in 1 kg of water.

Solution:
$E = (1\ \text{kg}) \times (3 \times 10^8\ \text{m/s})^2$
$= 9.0 \times 10^{16}$ J

This is enough energy to power 1 million, 60 W light bulbs continuously for 48 years!

9. Conservation of Energy

So, there are many different forms of energy. The most important thing about energy is that it is neither created nor destroyed -- it just changes form. This is one of the most important laws of nature. Thus, if a force performs work on an object the work done either raises the kinetic energy of the object, its potential energy, or the work goes into heat (or some combination of the above). Keeping track of the energy has been referred to as "energy accounting" because it is analogous to the bookkeeping performed by an accountant who keeps track of the money.

Processes that convert energy from one form to another often involve some friction. For instance, if the water stored in a water tower was allowed to fall so that it drives an electric generator, the total electric energy produced would probably by less than the gravitational potential energy originally stored in the water. Let E_f be the electrical energy produced and E_i be the initial potential energy of the water. The efficiency, η, of this process for producing electrical energy is defined to be

$$\eta = \frac{E_f}{E_i} .$$

If any of the initial energy is transformed into heat during this process then the efficiency will be less than 100%. If your goal is to convert kinetic or potential energy into heat -- this can be a 100% efficient process! But all processes which convert energy from one kinetic/potential form to another necessarily generate some unwanted heat along the way, and so, have efficiencies less than 100%. As technology is perfected the efficiency of these processes can approach 100%.

It is also possible to convert heat into work (i. e., other forms of energy) -- this is, in fact, what the purpose of an engine. The efficiency of an engine is defined to be the ratio of the work out to the heat in. As you will see in our next chapter, the 2nd law of thermodynamics places fundamental limitations on the efficiency of an engine which cannot be overcome by improved technology.