

PROBLEM SHEET 1

Which problems are actually due on a particular Friday will be announced at the end of lecture on the preceding Monday. This information will also be posted to the course web site, www.oberlin.edu/math/faculty/wilmer/343.

Since Cameron groups all the questions for his rather long chapters together, I will provide suggested reading intermittently, between groups of problems.

Reading. Sections 3.2, 3.3, and 3.4 of Cameron.

Problems from Cameron. Chapter 3, pages 44–48, problems 4a, 4d, 4e, and 7. (You should give combinatorial proofs for 4a and 4d if at all possible. Also, notice the hint for 4e on page 339.)

Additional problem A. Let x be a real number and let k be a natural number. Define the k -th factorial power of x by

$$x^{\underline{k}} = x(x-1)\cdots(x-k+1).$$

(a) Prove that factorial powers satisfy the following analogue of the binomial theorem:

$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^{\underline{k}} y^{n-k}$$

(A combinatorial proof is, of course, preferred, although there are certainly others out there!)

Additional problem B. Let $F_0 = 1, F_1 = 1, F_{n+1} = F_n + F_{n-1}$ for $n \geq 0$ be the Fibonacci numbers. Prove that

$$F_n = \sum_k \binom{n-k}{k}.$$

Reading. Sections 3.5 through Section 3.8, and Section 3.11, of Cameron.

Problems from Cameron. Chapter 3, pages 44–48, problems 10, 12, 14, and 21.

Additional problem C. Although these questions demand numerical answers (and you should provide them—the numbers are large enough that a calculator might be helpful), be sure to explain the reasoning behind your arithmetic.

- (1) How many permutations π of $\{1, 2, \dots, 7\}$ satisfy both $\pi(1) \neq 1$ and $\pi(2) \neq 1$?
- (2) How many permutations π of $\{1, 2, \dots, 7\}$ satisfy both $\pi(1) \neq 1$ and $\pi(2) \neq 2$?
- (3) How many permutations of $\{1, 2, \dots, 7\}$ have exactly two cycles?