# FINAL PRESENTATIONS

## GUIDELINES FOR YOUR PRESENTATION

- You should work in pairs. However, because we have an odd number of students in the class, I will allow (and hope to find) one group of three students.

- Each group's presentation will take up one half of one class period, i.e. 25 minutes. We will have presentations on December 5, December 8, and December 10. Scheduling will be done on a first-come, first-reserved basis.

- Just under half an hour is not a long time to speak on a mathematical subject. What should fill that time: a focussed, well-prepared discussion of a subject related to the course, one that is comprehensible to and informative for your fellow students.

  It's likely that some presentations will give detailed accounts of a particular example or construction, while others will give overviews of a broader area.

  Much of the work that you will do in preparation will consist of figuring out what material, from what you've read and understood, you want to present, and how best to explain it.

- Before your presentation, you will:
  - turn in a preliminary outline of the presentation,
  - turn in a detailed outline of the presentation, and
  - rehearse the presentation for me in the Math Library.

  The preliminary outline should be turned in at least two weeks before your presentation. The detailed outline should be turned in at least a week before your presentation. The rehearsal should take place at least 40 hours before your presentation. (Those presenting Monday should expect to rehearse on Friday afternoon.) Each of the outlines and your rehearsal will carry some weight in your grade for the presentation.

- You should let me know who you'll be working with, and what topic you will examine, by Wednesday, November 12.

## SUGGESTED TOPICS

I have references (usually some combination of sections in other textbooks and *American Mathematical Monthly* articles) for all of the topics below; come in and ask!

If there is anything that's come up in the course that you think you might like to study further, please let me know; the list below is certainly not exhaustive.

Some of you have already spoken with me about possible topics. I have included these topics on the list below, but with asterisks.

**Rings.**

*Quaternions and octonions.* These non-commutative (and, in the case of octonions, non-associative) rings, while no longer as central as their nineteenth-century discoverers hoped, have fascinating properties and more applications (to video-game programming? thanks to relations with 3-D rotation matrices) than you'd expect.

*Localization*.* By building in multiplicative inverses for all non-zero elements outside a maximal ideal, we get a ring with only one maximal ideal. Sometimes it's possible to prove things about the original ring by analyzing all of its localizations—which are simpler rings.

*Ideal factorization.* What happens when unique factorization fails? Ideals are called “ideals” because there’s a notion of ideal factorization, one which often works better than factorization for elements. (Artin’s textbook has a very nice development of ideal factorization in rings of quadratic integers, whether real or imaginary; there’s probably enough there for a full hour, if there are two interested pairs.)

*Wedderburn’s theorem.* Every finite division ring is a field. There are many accessible proofs: take your pick!

*Modules.* Just as there are vector spaces over fields, there are modules over rings. Because rings have much more diverse structures than do fields, modules are not so easily characterized as are vector spaces.

*Reasonable rings with bad properties.* There are a lot of them out there, lurking just out of view.

**Fields.**

*Constructibility, given more tools.* For example: which regular $n$-gons could you construct, if you did have an angle trisector?

*Transcendence bases.* What can you say about the structure of $\mathbb{Q}[e, \pi]$? Just like finite extensions have finite linear bases, the number of algebraically independent elements is an invariant of an extension.

*Inseparable extensions.* What happens when irreducibles can have repeated roots?

*Inverse questions in Galois theory.* Which groups arise as Galois groups of extensions, and how are constructions with those Galois groups constructed?

*How to compute Galois groups.* Given a polynomial, what can you do to determine what the Galois group of that polynomial really is?

**Other.** These are topics that really live in analysis or number theory, but have significant connections to this course.

*Transcendence of $e$ and $\pi$.* These aren’t so bad to prove; it does take some calculus, though.

*Approximability of irrationals by rationals.* Going a bit further than we did in lecture lets you show that $\phi = \frac{1+\sqrt{5}}{2}$ is the worst-approximable number.

*Differential Galois theory*.* You may have heard that the function $e^{-x^2/2}$ can’t be integrated in terms of elementary functions. What does that mean, and how is it proved?