

ASSIGNMENT 3

DUE FRIDAY, SEPTEMBER 19

Reading. Sections 3.5, 3.6, and 3.7 of Herstein.

Herstein problems.

- Section 3.6, problems 4 and 6.
- Section 3.7, problem 3, 7, and 8 (you may give a combined proof of 7 and 8, if it seems appropriate to do so).

Two additional problems.

1. Let M be the ring of 2 by 2 matrices with real entries. Prove that every non-zero element of M is either a unit or a zero divisor.
2. Let m be a positive integer and let

$$R = \mathbb{Z}[\sqrt{-m}] = \{a + b\sqrt{-m} \mid a, b \in \mathbb{Z}\}.$$

Define $N : R \rightarrow \mathbb{Z}$ by

$$N(a + b\sqrt{-m}) = a + mb^2.$$

- (a) Prove that, for $r, s \in R$, $N(rs) = N(r)N(s)$.
- (b) Find all the units in R . Prove that your answer is correct.