MATHEMATICS 329—RINGS AND FIELDS—NOVEMBER 14, 2003

ASSIGNMENT 10

DUE FRIDAY, NOVEMBER 21, 2003.

Reading. Handouts from Artin.

Problems.

1. Prove that the discriminant of a cubic polynomial with real coefficients is positive if all the roots are real, and negative otherwise.

2. Let δ be a square root of the discriminant of an irreducible cubic $x^3 + px + q$, whose roots are $\alpha_1, \alpha_2, \alpha_3$. Give a formula for α_2 in terms of α_1, δ, p, and q.

3. Let $f(x) \in \mathbb{Q}[x]$ be an irreducible cubic polynomial with exactly one real root. Prove that the Galois group of $f(x)$ is S_3.

4. Let K be the splitting field of $x^4 - 3$ over $\mathbb{Q}$.

   (a) Prove that $[K : \mathbb{Q}] = 8$ and that K is generated by i and a single root α of the polynomial.

   (b) Prove that the Galois group of $x^4 - 3$ is dihedral, and describe the operation of the elements of the group on the generators of K explicitly.

5. Determine the possible Galois groups of a *reducible* quartic of the form $x^4 + bx^2 + c$, assuming that the quadratic $y^2 + by + c$ is irreducible.

6. Can the roots of the polynomial $x^4 + x - 5$ be constructed by ruler and compass?