THREE MATHEMATICAL CURIOSITIES

What's right about these three "proofs"? More importantly, what's wrong?

Theorem 1.
$$\frac{1}{0} = \infty.$$

Proof. Recall the well-known fact that
$$\frac{1}{\infty} = 0.$$
Rotate both sides of the equation 90 degrees counterclockwise, obtaining
$$-18 = 0.$$
Next, add 8 to both sides:
$$-10 = 8.$$
Now simply rotate both sides of the equation clockwise by 90 degrees:
$$\frac{1}{0} = \infty. \qquad \square$$

Theorem 2. *All horses are the same color.*

Proof. More precisely, we prove that *for any natural number n, every collection of n horses consists of horses of a single color.*

When $n = 1$, any collection under consideration consists of a single horse, so the claim is clearly true.

Now, assume the claim is true for some specific value N. We will demonstrate that it must then be true for $N + 1$ as well. Consider an arbitrary collection of $N + 1$ horses:

$$h_1, h_2, \ldots, h_{N+1}.$$

By our assumption, the subcollection $h_1, \ldots, h_N$, which contains N horses, is monochromatic. By the same reasoning, the subcollection $h_2, \ldots, h_{n+1}$, which also contains N horses, is monochromatic.

We can conclude that h_1 and h_{N+1} are both the same color as all the horses in the subcollection $h_2, \ldots, h_N$. Thus all $N + 1$ horses are the same color. □

Theorem 3. *Every natural number can be described unambiguously in English using sixteen or fewer words.*

Proof. Suppose there is some natural number which cannot be unambiguously described in English in sixteen or fewer words. Then there must be a smallest such number. Let's call it n.

But now n is "the smallest natural number that cannot be unambiguously described in English in sixteen or fewer words." This is a complete and unambiguous description in English of n using exactly sixteen words, contradicting the fact that n was not supposed to have such a description!

Since our initial assumption of the existence of a natural number that cannot be unambiguously described in English using at most sixteen words led to a contradiction, it must be an incorrect assumption.

Therefore, all natural numbers can be unambiguously described in English in sixteen or fewer words! □