MATHEMATICS 220—DISCRETE MATHEMATICS—NOVEMBER 6, 2006

ASSIGNMENT 8

DUE FRIDAY, NOVEMBER 10.

Reading. Biggs, Chapter 8.

Problems.

(1) Prove that if the integers a, b, c, and d satisfy $a|b$ and $c|d$, then $ac|bd$.

(2) What is 700 (decimal) in binary? What is 12121 (base 3) in decimal?

(3) Find (a) $\gcd(123, 277)$. (b) $\gcd(1529, 14039)$.

(4) Proof or counterexample:

(a) For all positive integers n, $\gcd(2n - 1, n) = 1$.
(b) For all positive integers n, $\gcd(4n - 2, n) = 2$.

(5) Find integers x and y such that $\gcd(323, 124) = x(323) + y(124)$.

(6) Prove that a positive integer n, $n > 1$, is a perfect square if and only if when we write

$$n = p_1^{e_1} p_2^{e_2} \dots p_r^{e_r}$$

with each p_i prime and $p_1 < \cdots < p_r$, every exponent e_i is even. (Hint: use the fundamental theorem of arithmetic!)