MATHEMATICS 220—DISCRETE MATHEMATICS—OCTOBER 31, 2006

ASSIGNMENT 7

DUE FRIDAY, NOVEMBER 3.

Reading. Biggs, Sections 7.1–7.5.

Problems.

(1) Which of the following relations on the set of all people are equivalence relations? For those which are equivalence relations, describe the equivalence classes.

For those which are not equivalence relations, determine which of the defining properties of an equivalence relation are lacking.

In the following, you may assume that a and b always represent people.

(a) $\{(a, b) |\, a$ and b have the same birthday$\}$.

(b) $\{(a, b) |\, a$ is older than $b\}$.

(c) $\{(a, b) |\, a$ and b have the same parents$\}$.

(d) $\{(a, b) |\, a$ and b have at least one grandparent in common$\}$.

(2) Prove that, for the equivalence classes $[a, b]$ and operations $+$ and $\times$ on those classes defined in Section 7.4:

(a) For any $n, a, b \in \mathbb{N}$, $[n + 1, 1] \times [a, b] = [na, nb]$.

(b) The distributive law for $\times$ over $+$ is true.

(3) Prove that when x, y, and z are integers, then $x < y$ and $y < z$ together imply $x < z$. (You should use the definition of $<$ in terms of representatives of equivalence classes at the end of Section 7.5.)