MATHEMATICS 220—DISCRETE MATHEMATICS—SEPTEMBER 22, 2006

ASSIGNMENT 4

DUE FRIDAY, SEPTEMBER 29.

Reading. Sections 4.1–4.5 of Biggs, along with “Comments on Writing Proofs” handout.

Definitions.

Standard terms.

- Define at natural number n to be *even* if there exists a natural number k such that $n = 2k$.
- Define a natural number n to be *odd* if there exists a natural number k such that $n = 2k + 1$.
- Define a natural number n to be *prime* if and only if $n > 1$ and, whenever $n = rs$, where r and s are natural numbers, then $r = 1$ or $s = 1$.
- Define a natural number n to be *composite* if and only if there exist integers r and s satisfying $r > 1$ and $s > 1$ such that $n = rs$.

Terms specific to this class.

- Define a natural number n to be *treven* if and only if there exists a natural number k such that $n = 3k$.
- Define a natural number n to be *trodd* if and only if there exists an natural number k such that $n = 3k + 1$.
- Define a natural number n to be *negtrodd* if and only if there exists a natural number k such that $n + 1 = 3k$.
- Define a natural number n to be *choice* if and only if there does not exist a natural number m such that $m > 1$, $m < n$, and n is a multiple of m.

Problems.

(1) Prove that there exists a number which is both odd and trodd.

(2) Prove that the sum of three trodd numbers is treven.

(3) Prove that every treven number greater than 3 is composite.

(4) Prove that a natural number n is choice if and only if it is prime. (Hint: you may find it easier to prove contrapositives.)

(5) Some of the following statements are true and some are false. Prove the true ones and give counterexamples for the false ones.

   (a) The sum of two trodd numbers is trodd.

   (b) The product of two negtrodd numbers is trodd.

   (c) The sum of a trodd number and a negtrodd number is treven.

   (d) The product of a trodd number and a negtrodd number is treven.

(6) Prove, using induction, that for every natural number n,

$$3(1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1)) = \sum_{i=1}^{n} 3i(i+1) = n(n+1)(n+2).$$

(7) Define, for natural numbers a and n,

$$a^1 = a \quad \text{and} \quad a^{n+1} = a^n \cdot a.$$

Prove, using induction, that for every natural number $n \geq 6$,

$$3^n < n!$$