

# ASSIGNMENT 3

DUE FRIDAY, SEPTEMBER 22.

**Reading.** Sections 3.1–3.6 of Biggs.

**Problems.**

- (1) Determine whether  $p \rightarrow (q \rightarrow r)$  and  $(p \rightarrow q) \rightarrow r$  are logically equivalent.
- (2) Let  $|$  be the Sheffer stroke operation defined in lecture, with truth table

$p$	$q$	$p q$
$T$	$T$	$F$
$T$	$F$	$T$
$F$	$T$	$T$
$F$	$F$	$T$

Construct compound propositions, using only  $|$ , that are logically equivalent to

- (a)  $\neg p$ .
- (b)  $p \vee q$ .
- (c)  $p \wedge q$ .

(You should include the truth tables that justify your answers.)

- (3) Determine whether each of the following is a valid or invalid argument. Include the truth tables that justify your answers.

(a) $\frac{p \rightarrow q}{q \rightarrow r} \quad \therefore p \rightarrow r$	(b) $\frac{p \rightarrow (q \vee r)}{\neg r} \quad \therefore \neg p$
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- (4) Let  $L(x, y)$  be the statement “ $x$  loves  $y$ ”, where the universe of discourse for both  $x$  and  $y$  is the set of all the people in the world. Use quantifiers and the predicate  $L$  to express each of the following statements:

- (a) Everybody loves Jerry.
- (b) Everybody loves somebody.
- (c) There is somebody whom everybody loves.
- (d) Nobody loves everybody.
- (e) There is somebody whom no one loves.
- (f) There is exactly one person whom everybody loves.

- (5) Determine the truth value of each of the following statements, given that the universe of discourse for all variables is the set  $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$  of integers. Include a sentence or two of explanation of each answer.

- (a)  $\forall n((n^2 \leq n) \rightarrow (n \leq 1))$
- (b)  $\forall n \exists m(n^2 < m)$
- (c)  $\exists m \forall n(m < n^2)$
- (d)  $\forall m \forall n \exists p(p = \frac{n+m}{2})$
- (e)  $\forall m \exists n(nm = 1)$
- (f)  $\exists m \forall n(nm = m)$

- (6) Define  $\exists! x P(x)$  to be true exactly when there is a unique value of  $x$  for which the predicate  $P(x)$  is true. In the examples below, take the universe of discourse for all variables to be the integers.

- (a) Is  $\exists! n(n^2 = 4)$  true or false?
- (b) Is  $\exists! n \forall m(nm = 0)$  true or false?
- (c) Construct an expression, using only the predicate  $P$ , along with standard existential and/or universal quantifiers and predicate logic operators ( $\neg, \wedge, \vee, \rightarrow$ ), that is logically equivalent to  $\exists! x P(x)$ .