**MATH 328** **Fall 2020**

# Computational Algebra and Algebraic Geometry

**MWF 11:20 – 12:10 pm King 106**

**Instructor:** Susan Jane Colley
King 222
775-8388 (office) or -8380 (messages)
440-823-0887 (cell phone—please call before 10:00 pm)
E-mail: scolley@oberlin.edu
sjcolley@math.oberlin.edu
Web page: www.oberlin.edu/math/faculty/colley.html

**Office Hours:**

| | |
|---|---|
| Monday | 3:30 – 5:00 pm |
| Tuesday | 10:00 – 11:00 am, 3:00 – 4:30 pm |
| Wednesday | 3:30 – 4:30 pm |
| Thursday | 10:00 – 11:30 am |
| Friday | 2:00 – 3:00 pm |

Also by appointment

Zoom meeting link for office hours: https://oberlin.zoom.us/j/669553878

**Text:** D. Cox, J. Little, and D. O'Shea, *Ideals, Varieties, and Algorithms*, 4th. ed., Springer. This text is required and is available at the bookstore. It is also available electronically at https://doi.org/10.1007/978-3-319-16721-3. (Electronic access is by authentication with the Oberlin library proxy server or VPN.)

**Goals:**

- Explore topics from commutative algebra (particularly ring theory) and elementary algebraic geometry (i.e., varieties).
- Examine both theory and examples in concrete terms through algorithms enabling computer computations within abstract algebra.
- Understand how some key algebraic ideas can be applied to other areas in the mathematical sciences.
- Learn additional, related topics through independent work.
- Practice technical mathematical writing and speaking.

**Homework:** There will be weekly, hand-in problem sets during the semester. You may work together on this homework, but you must submit your own write-up of the problems and indicate with whom you had any significant collaboration. No late assignments will be accepted (emergencies excepted, of course). Solutions to the hand-in problem sets will be available online through Blackboard. Note that homework will involve *peer-grading*. See page 5 for more information about how to submit homework and how it will be graded.

**Exams:** There will be two open-book, take-home exams, tentatively **due on Wednesday, October 7 and Wednesday, November 11**. Please let me know as soon as possible if there is a problem with either of these dates. There will also be a final exam scheduled for **Sunday, December 13 from 2:00 to 4:00 pm**.

**Project:** You will work with one or two other students on an independent project related to computational algebra. The project may involve material from the text or from other sources, or it may represent independent work. Each group

will submit a written report (~ 5–7 pages) concerning the project and make a short presentation **to be scheduled between December 2 and 5**. Additional details concerning the special project will be forthcoming.

**Participation:** It is important that you come to class—on time and for (nearly) every meeting. Further, you should attempt to participate actively in class by asking questions, offering problem solutions, etc.

**Grading:**

| | |
|---|---|
| Take-home midterms (each) | 20% |
| Final exam | 20% |
| Homework | 20% |
| Special project | 20% |

(percentages of final grade)

**Deadlines:** I will try to be as clear as I can about the nature of assignments, and I will provide fair warning about when they are due. **Late assignments normally will not be accepted.** At the same time, I do understand that emergencies arise, so if *unforeseen* circumstances are interfering with your ability to complete some work in the course (e.g., significant illness, but *not* assignments for other classes), please contact me immediately, preferably *before* the assignment is due.

**Online:** Copies of assignments, handouts, etc. will be posted on the course Blackboard site. Go to **`blackboard.oberlin.edu`** (and your "Academic Hub") to access these materials.

**Note:** If you have a documented disability I am happy to discuss academic accommodations with you; please contact me as soon as possible so I can understand how to be of assistance.

## Outline of the Course

| | |
|---|---|
| Introduction and basic notions (Chapter 1) | 2.5 weeks |
| Gröbner bases (Chapter 2) | 3.5 weeks |
| Elementary elimination theory (Chapter 3) | 2.5 weeks |
| Correspondence between ideals and varieties (Chapter 4) | 3 weeks |
| Additional topics | as time permits |

# Selected Bibliography

Here are some suggestions for supplementary reading. You may find some of these references helpful for putting together your project, but you should not feel constrained by them.

## BOOKS


W. W. Adams and P. Loustaunau, *An Introduction to Gröbner Bases,* American Mathematical Society, Providence, 1994.

T. Becker and V. Weispfenning, *Gröbner Bases: A Computational Approach to Commutative Algebra,* Springer, New York, 1993.

D. Cox, J. Little, and D. O'Shea, *Using Algebraic Geometry,* Springer, New York, 2nd ed. 2005. (This book is available electronically through OBIS.)

D. Eisenbud, *Commutative Algebra with a View Toward Algebraic Geometry,* Springer, New York, 1995.

V. Ene and J. Herzog, *Gröbner Bases in Commutative Algebra*, American Mathematical Society, Providence, 2012.

R. Fröberg, *An Introduction to Gröbner Bases*, Wiley, New York, 1997.

N. Koblitz, *Algebraic Aspects of Cryptography,* Springer, New York, 1998.

H. Schenck, *Computational Algebraic Geometry,* Cambridge, 2003

J. van Lint and G. van der Geer, *Introduction to Coding Theory and Algebraic Geometry,* Birkhäuser, Basel, 1988.


## PAPERS/ANTHOLOGIES


D. Bertsimas, G. Perakis, and S. Tayur, "A new algebraic geometry algorithm for integer programming," *Management Science* **46** (2000), no. 7, 999–1008. (I have a copy of this paper.)

B. Buchberger and F. Winkler, eds., *Gröbner Bases and Applications,* Cambridge University Press, Cambridge, 1998.

P. Conti and C. Traverso, "Buchberger algorithm and integer programming," in *Applied Algebra, Algebraic Algorithms and Error-correcting Codes (AAECC-9),* Lecture notes in Computer Science **539**, Springer, New York, 1991, 130–139. (I have a copy of this paper.)

D. Cox and B. Sturmfels, eds., *Applications of Computational Algebraic Geometry,* American Mathematical Society, Providence, 1997.

D. Eisenbud and L. Robbiano, eds., *Computational Algebraic Geometry and Commutative Algebra,* Cambridge University Press, Cambridge, 1993.

# Honor Code Policies



## Homework

You are permitted, even encouraged, to collaborate on homework. For homework that is not graded, feel free to consult anyone at all: your classmates, me, other students, friends, relatives, Donald Trump (this last one not really). For homework that is to be handed in and graded, I expect you to be somewhat more careful. Specifically, you should continue to ask questions of me regarding homework problems and you may collaborate with one or two of your classmates (per assignment). Please do not undertake significant collaboration with more than two students without permission. If you do collaborate, you are expected to write your own solution to problems (i.e., not to copy) and to indicate the name(s) of any student(s) with whom you worked.

You may consult any written sources for hand-in homework, provided that you give appropriate citations. Please write your homework solutions with care.

## Examinations

Unless specifically indicated otherwise, "in-class" tests are assumed to be closed-book. Collaboration of any sort (other than to ask me questions) will **not** be permitted. Take-home exams will have specific provisions for using books and notes, but, again, you are **not** to discuss the content of the exam with anyone other than me. Any time limits will be indicated with each test.

## Independent Project

Obviously, your primary contacts for your project should be me and your partner(s). It is also acceptable for you to consult with others, provided you give appropriate citation in your presentation and your written report.

## Honor Pledge

On every assignment that you submit for credit, you are expected to sign the Oberlin College Honor Pledge:

"I have adhered to the Honor Code on this assignment."

If you need clarification of the policies above, please do not hesitate to ask. Should you require some variation in these rules, you must discuss the matter with me well in advance of any assignment.

For general information about the Honor System at Oberlin, consult

https://www.oberlin.edu/dean-of-students/student-conduct/academic-integrity/students.

# More About Homework

## Submitting Homework

You will turn in your problem sets in electronic form, as pdf files. If you do not plan to use LaTeX (or Word), then please **download a scanner app on your phone**; there are several free scanner apps available which *can scan documents and create pdf files*. After downloading the app, please scan your solutions to each problem set with your phone and create a **single pdf file**. **Please check that all your work is clearly legible**; scanner apps are powerful enough not to have a problem with this, but dark, clear writing (for example, with a pen rather than a pencil) is important. Since we will undertake peer grading of homework (see below), ***your name should not appear anywhere in the contents of the pdf file that you submit.*** (I will return your homework to you if your name appears in it.) However, please **name your pdf file in the following way:**

> **PS1_LastName_FirstName.pdf** (so I would name my file PS1_Colley_Susan.pdf; please note that these are *underscore characters, not dashes*)

I will (temporarily) rename your file with a code for the purpose of peer grading, in order to protect your privacy.

When you email me your homework, you should also always do the following:

1. Use a subject header of the following form: Problem Set <xx> from <your name>.
2. Accompany your problem set attachment with a message indicating the names of any collaborators and a written statement that you adhered to the Honor Code:

   Attached is Problem Set XX. I collaborated with A and B in developing the solutions.
   I have adhered to the Honor Code on this assignment.
   Typed signature.

## Peer Grading of Homework

As you know, clear and effective technical writing is a goal of every mathematics course, particularly at this level. One way that you can improve your own mathematical writing is to review and comment on the writing of others. Thus, twice during the semester, along with other students, you will work with me to grade homework. This is a mandatory activity, but I will provide lists for you to volunteer to grade for a particular week.

**Expectations:** This activity should involve a commitment of no more than two hours (for each grading session). I will supply you with solutions and some initial guidance regarding partial credit. You will work through the papers you are assigned; what is important is that you provide some constructive comments as you reader through the papers. You can make these comments using Adobe Acrobat Reader by means of sticky notes. Thus, please be sure to download a copy of Adobe Acrobat Reader if you do not already have one.

After the grading is completed, each peer grader should email me a brief feedback report, reflecting on the process (e.g., an especially interesting argument or solution that you observed, or a frequent problem with a solution).

You will receive 5 homework points for each of the peer grading sessions in which you participate and submit a feedback report.