(Draft version.)

Models, the Semantic View, and Scientific Representation[+]

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Introduction

Talk of models, though certainly not a novel feature of the philosophy of science of recent years, has become more and more central to the discussion of a good many topics in the field, not least amongst them the intertwined topics of today's symposium: theory structure, the nature of scientific representation, and the defensibility of some version of structural realism. Unfortunately, however, the word 'model' has become multiply ambiguous along the way. There is a profusion of different notions abroad which go by that name, and a serious danger that the number will continue to grow unchecked. It is easy to be led astray in such a situation, by moving in too carefree a way between importantly different notions of model, and the main aim of my talk today is to describe one way in which I think that, indeed, we have gone astray. More specifically, I want to describe a way in which I think the so-called semantic view of scientific theory structure has often been misunderstood, and lay out what I take to be the right way of understanding it – or, at any rate, the best way. I will close with a few suggestions about the implications for structuralism about scientific representation, and for structural realism.

A taxonomy

Let me begin by presenting, in summary form, a taxonomy of models.[1] I'll list the five categories making up the taxonomy with sample authors who have articulated notions of model which are least closely related.

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1) Truth-making map: A model as a mapping from parts of a language which provides an interpretation for, and makes true, some given set of sentences in that language. (Enderton)

2) Truth-making structure: A model as a (generally) nonlinguistic structure which provides an interpretation for, and makes true, some set of sentences. (Tarski)

3) Mathematical model: A model as a mathematical structure used to represent a (type of) system under study. (Suppes, van Fraassen)

4) Propositional model: A model as a set of propositions, the members of which together form a representation of a (type of) system under study. (Achinstein, Redhead)

5) Physical model: A model as an actual, concrete physical object used to represent a (type of) system under study. (Boltzmann)

I will flesh out the characterisations of the first three categories in a moment, as they are the ones which are directly relevant to the points I wish to make about the semantic view. But it will be useful to make two more general points about this taxonomy before getting to grips with the details.

First, it is immediately apparent that each of the five senses of 'model' I have distinguished can be thought of as having two components: To call something a model, in each of these five senses, is partly to classify it as a certain sort of thing, and partly to ascribe a certain role to it.[2] There is a job to do, and a kind of thing doing it.

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[1] For a fuller presentation, especially with regard to the fourth and fifth categories, see Thomson-Jones (forthcoming b). For earlier attempts at the same project, see Suppes (1960), Suppe (1967), and Achinstein (1968). See also Lloyd (1998) and Frisch (1998).

[2] Of course, the phrase "partly to classify it as a certain sort of thing" flirts with emptiness. If it were important to make this characterization more concrete, we might try something like "partly to classify it with respect to its intrinsic (i.e., nonrelational) properties," where the implicit idea would then be that playing a certain role is (at least in these cases) a relational property. Consideration of the specific examples at hand should suffice to make my meaning clear, however.

Approached from the point of view of this formula, it becomes clear that the five notions of model can be divided into two groups. The first two notions can be grouped together because in both cases a model is something which, in one way or another, does the job of providing an interpretation for a certain set of sentences, and in such a way that those sentences come out true. To be a model in either of these senses is thus to be a model of a set of sentences made true. To be a model in the third, fourth, or fifth sense, however, is in part rather to do the job of representing a system, or type of system, from the domain of inquiry (whether actual or merely possible)[3]; to be a model in these latter senses is thus, in part, to be a representation. Given this grouping, then, we might abstract out from my fivefold classification two overarching notions of model, namely, the notion of model as truth-maker, and the notion of model as representation.[4] (To characterize this dichotomy in terms of a more familiar philosophical dichotomy, we might say that the two most general notions of model at play here can be distinguished by the direction in which the 'of' in 'model of' points: in one case, to language, and in the other, to the world.) For many purposes, this relatively coarse-grained distinction between models as truth-makers and models as representations may be all we need. Some of my reasons for recommending a slightly more complex taxonomy nonetheless will become clear as we go along.[5]

The second general point I want to make about the taxonomy is that it is intended to be comprehensive, but only in one specific sense, and even then not without qualification. As to the sort of comprehensiveness aimed at: Each of the five basic notions making up the taxonomy can be found in one place or another in the philosophy of science (and, in some cases, in logic, or in science itself), and various other notions

[3] Although not necessarily with any veracity.

[4] For an extensive discussion of this distinction and its importance in this context, see Frisch (1998). I am indebted to Mathias Frisch for numerous very helpful conversations.

[5] One clarification regarding my choice of terminology in the taxonomy: As it turns out, the word 'model' appears only in labels corresponding to notions of model which involve representation as a crucial component. No significance should be attached to this; in particular, I do not mean to suggest that the term 'model' should really only apply to objects which function as representations.

employed in the literature pick out special subclasses of one of the five corresponding classes of model, or are closely related to one of the five basic notions in some other way. Nonetheless, the taxonomy is not a taxonomy of *notions* of model, and there are certainly important notions of model around which are not in any sense included amongst the notions making up my taxonomy. My hope is nonetheless that all the things philosophers of science talk about when they use the term 'model' are models in one or more of the senses covered in my taxonomy – they are, in other words, objects which belong in one or more of the five categories making up the taxonomy I have just presented.[6] As to the qualifications needed: There is at least one notion in the literature which does not pick out models in any of the five senses given above, but it is not a notion which is currently in active service.[7] There are also one or two other notions of model which do not *seem* to pick out models in any of these five senses, but which are certainly current – I am thinking primarily of Giere's central notion of model, and at

[6] One example, to illustrate the point, would be Morrison's notion of a mediating model [[insert ref., and check attr.]]. The notion is a notion which relates straightforwardly to any of the notions employed in laying out my taxonomy, but mediating models, I would claim, are all either mathematical models or propositional models.

[7] I have in mind a notion of model to be found in the work of some proponents of the Received View of theories. According to the Received View, of course, a theory should be thought of as consisting, first and foremost, of a set of uninterpreted sentences in a formal language, possibly accompanied by a set of syntactically defined rules for deriving such sentences from one another. Authors such as Nagel (1961), Braithwaite (1962), and Spector (1965) then used the term 'model' to denote a set of *interpreted* sentences, derived from such a set of uninterpreted sentences by the superaddition of an interpretation for each of the non-logical terms. (There are some differences between authors with regard to the additional conditions to be placed on a model in this sense, but those differences are largely irrelevant here.)

Now one might regard this sentential notion of model as closely akin to the notion of a propositional model; for some purposes, after all, we can regard an interpreted sentence and the proposition it expresses as interchangeable, and a set of interpreted sentences the members of which make claims about the hydrogen atom can surely be regarded as a representation of that kind of system. Unfortunately, things are not so neat. For the authors in question, models in this sentential sense are models of theories, not models of systems from the domain of inquiry. These authors call these sets of interpreted sentences "models" because of a relation the sets stand in to theories, a relation which is clearly centrally to do with interpretation, and, in Nagel's case, making true; they are not so-called because of any representation relation they might stand in to gases, or economies, or chemical processes. In many respects, then this notion of model seems to belong with the first and second notions we discussed. However, models in this sense are not objects of quite the right sort to be models in either of those senses – they are neither mappings nor structures mapped to. So here is one notion of model found in the philosophy of science which does not fit into my five-part taxonomy. I have chosen not to create a separate category, however, mainly for the reason that this notion of model has, by and large, fallen into disuse. It is too closely tied to the Received View of theories, and had its heyday when empiricist problems about the meaning of theoretical terms and about the logic of analogical inference in theory development predominated, and were framed in terms of that approach to theory structure.

least one of Suppe's. I argue elsewhere, however, that on close examination it turns out that those notions either fall into one of the five categories after all, or are incoherent.[8]

So much for the taxonomy as a whole. In order to be able to see the points I want to make about the best way of understanding the semantic view, we will do best to examine the second and third notions – of truth-making structures, and of mathematical models – more closely. And perhaps the easiest way of being clear-headed about the second is to start with the first.

Truth-making and representation

Truth-making maps. We begin by recapping a familiar use of the term 'model' drawn from logic. Suppose we have a set of sentences in a first-order language whose non-logical symbols are, in an intuitive sense, initially uninterpreted. Then a *structure* (also called an *interpretation*) for the language is a function which (i) maps the universal quantifier to a nonempty set, called the *universe* (or *domain of discourse*), (ii) maps each individual constant to an element of the universe, and (iii) maps each *n*-place predicate to a set of *n*-tuples of elements of the universe (i.e., to a subset of the *n*-fold Cartesian product of the universe with itself). Assuming that a formal definition of the notion of truth in a structure has been provided, a *model* of the set of sentences in question is then simply a structure on which all the sentences of the set come out true.[9]

A simple example will prove useful in contrasting this sort of usage of the term 'model' with the next. Suppose we are dealing with a rather impoverished first-order language, one which contains no constants (or function symbols) and just two predicates, 'P' and 'Q', each of which is unary. Then one structure for this language maps the universal quantifier onto the set

$$S_1 \equiv \{\text{Posh, Ginger, Baby, Sporty, Scary}\},$$

[8] See Thomson-Jones (forthcoming a).

[9] See, e.g., Enderton (1972), pp. 79-84, for a fuller and more rigorous exposition. For simplicity's sake, I am supposing that we are dealing with a language which contains no function symbols. The extension to functions is simple enough; see Enderton (*loc. cit.*).

maps ‘P’ onto the set

$$S_2 \equiv \{\text{Posh}\},$$

and maps ‘Q’ onto the set

$$S_3 \equiv \{\text{Posh, Ginger}\}.$$

And on the standard definition of truth in a structure, this structure is a model of the set

$$\{(\forall x)(Px \rightarrow Qx)\},$$

for on that definition, ‘$(\forall x)(Px \rightarrow Qx)$’ is true in a given structure if and only if the extension of ‘P’ is a subset of the extension of ‘Q’.

We can then abstract from this specific use of the term ‘model’ the somewhat more general notion of a model as a mapping from parts of a language to objects of one sort or another, one which provides an interpretation for the sentences in a given set, and does so in such a way that they come out simultaneously true.[10] I will use the label ‘truth-making map’ to refer to models of this sort.

The notion of a truth-making map generalises from the logical notion specified above in at least two ways. First, it covers mappings which interpret languages other than the language of ordinary first-order logic. In constructing a semantics for modal logic, for example, we can again define a model to be a mapping from symbols in the language to various sets and members thereof, and one on which all the sentences in a given set come out true, even though the range of mapping will now have a more complex structure.[11] (And perhaps, too, sets of sentences of a natural language can have

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[10] Note that there are certain respects in which this characterisation of the general notion must be understood somewhat loosely if that notion is to apply to the models of an orthodox model-theoretic semantics for a first-order language: first, with regard to the notion of interpretation employed in the characterisation of the general notion, as (i) some of the symbols in the sentences of a first-order language (such as ‘→’) already have their meanings fixed, and (ii) it is a matter of controversy in metaphysics and the philosophy of language to what extent we succeed in attaching a meaning to (or “interpreting”) a predicate symbol merely by fixing its extension, and insofar as ‘P’ lacks a full interpretation, so too must any sentence in which it appears; second, with regard to the notion of “coming out true,” for strictly speaking the sentences in a set of first-order sentences only come out *true in a structure*, and the truth in a structure is not the same thing as truth.

[11] For a usage of this sort in the context of modal logic, see Goldblatt (1993). (I am indebted to Charles Chihara for this reference.) This is by no means a universal terminology, however. Hughes and Cresswell (1984, pp. 167-8), for example, define a model for a first-order modal language (without constants or functions) to be an ordered 3-tuple consisting of a set of worlds, a set of individuals, and a mapping from (i) the variables of the language to elements of the set of individuals, and (ii) the n-place predicate symbols to sets of n+1-tuples (consisting of n individuals and one world). So, although a model in their sense interprets

models in this truth-making map sense.) Second, it allows the interpretation of the symbols of a given language to proceed in less set-theoretical ways, such as by the provision of a set of "semantical rules" in a meta-language (e.g., "let 'P' mean the same as 'married David Beckham'").[12] A model will, in such a case, still be a mapping of the symbols in the language onto various objects in such a way as to provide an interpretation of the sentences in a certain set on which they all come out true, but the "objects" (the items in the range of the mapping) will now be fully interpreted terms in a metalanguage, or, more provocatively, the meanings of such terms.

Truth-making structures. In order to work our way towards the second general notion of model, let us return to the first kind of truth-making map discussed, the one exemplified by our example involving The Spice Girls. In that example, the model was a mapping from three symbols in the language in question ('∀', 'P', 'Q') to three particular sets. It is now a simple enough manoeuvre to focus our attention on the sets rather than on the mapping itself. More specifically, we might consider the ordered triple

$$\langle S_1, S_2, S_3 \rangle.$$

We then arrive at Tarski's notion of a model of a set of sentences of a first-order language, on which $\langle S_1, S_2, S_3 \rangle$ is a model of $\{(\forall x)(Px \rightarrow Qx)\}$, because the uninterpreted terms of the first-order language in which the members of $\{(\forall x)(Px \rightarrow Qx)\}$ are written can be mapped onto the elements of $\langle S_1, S_2, S_3 \rangle$ in such a way that all the members of $\{(\forall x)(Px \rightarrow Qx)\}$ come out true.[13]

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the formal language at hand, and although it contains a mapping as a part, it is not a mapping, and it is not an interpretation which makes a certain set of sentences true; it also provides denotations for the variables of the language, which is not something one of Enderton's models does.

[12] See, e.g., Spector (1965), pp. 276-277, n. 1 and n. 3, and his references to Carnap, Hempel, Nagel, Braithwaite, and Pap, to all of whom he attributes the notion of a semantical rule.

[13] See his (1953), p. 11, and (1956), p. 417. In order to see how this notion generalizes to the case of less minimal first-order languages, consider the following:

(i) We can always impose an ordering on the predicates, constants, and function symbols, of the language (if any), and then let that ordering determine the ordering of the elements of the tuple. Given such a procedure, it is easy to see that there will be a one-to-one correspondence between models of the mapping variety and models in the "tuple of sets" sense for any given set of sentences in any given first-order language.

Now the ordered triple <S_1, S_2, S_3> can be said to have a certain sort of internal structure, in that the second and third elements are subsets of the first, and the second is a subset of the third. Accordingly, we might call it a "truth-making structure."[14] And we can then abstract from this sort of talk the general notion of a model as a truth-making structure – that is, the notion of a model as a (generally nonlinguistic) structure which provides an interpretation for, and makes true, some set of sentences.[15]

Given our earlier discussion of the ways in which the notion of a truth-making map generalises from the most basic logical case, it is relatively easy to think of kinds of truth-making structure other than Tarskian models of sets of sentences of a first-order language. There will be truth-making structures for sets of sentences in other formal languages, such the languages of modal logic, and perhaps truth-making structures for sets of sentences of English, if the right sort of semantical theory of English is presupposed. And not all truth-making structures need be set-theoretical structures. Consider, for example, a tuple of fully interpreted terms from a metalanguage ('married David Beckham'), or of intensions, supposing there to be such entities, rather than a tuple of sets; the nonlogical terms in a first-order language (say) might be mapped onto the elements of such a tuple in such a way that all the members of some set of sentences in the first-order language come out true.[16]

(ii) Not all elements of such a tuple will, in general, be sets if we take constants to be mapped to elements of the universe of discourse, as is standard. I will assume, however, that the extension of a function symbol is a set.

(iii) It is standard to allow the predicates, constants, and function symbols to form a set of denumerably infinite cardinality. This simply means that we need to allow our ordered tuples to contain a denumerably infinite number of places.

[14] Confusion will ensue, of course, if we are not careful to separate this use of the word 'structure' from the use prescribed by the definition, discussed above, of a structure as a certain sort of mapping. A truth-making structure is *not* a mapping, but rather a thing mapped onto. (I might have tried to avoid this problem by opting for 'interpretation' rather than 'structure' as a label for the mappings, but that choice, too, carries a risk of confusion, as we also need to use the former term in other senses in the present discussion.)

[15] *Generally* non-linguistic because (a) there are truth-making structures for {$(\forall x)(Px \rightarrow Qx)$} of the <S, S', S''> variety in which the members of S, S', and S'' are, or include, linguistic items, and (b) if structured systems such as economies and electrical circuits can count as truth-making structures for certain sets of sentences once we broaden the notion sufficiently (see below), then presumably linguistic structures (such as languages) will count as truth-making structures for some sets of sentences.

[16] Of course, the notion of an ordered tuple is standardly given a set-theoretical definition, but (i) that fact plays no role here, (ii) the interpreted terms or the intensions could be arranged into a different sort of structure, such a mereological sum, and the terms of the first-order language then mapped onto various parts of the sum, and (iii) even if we do use tuples of interpreted terms, or of intensions, and understand the notion of a tuple set-theoretically, this sort of tuple is clearly not a set-theoretical structure in anything like

A further extension of the notion of a truth-making structure can be arrived at by a somewhat different route. Suppose that a certain system such as an electrical circuit, an organism, or the economy of a particular country provides the ur-elements of a set-theoretical structure which turns out to be a truth-making structure for some set of sentences in a formal language; perhaps, to invoke the simple Tarskian case, the universe of discourse (the first set in the tuple) has as its elements only parts of the system in question. Then we might, in a derivative way, speak of the system itself, the electrical circuit or economy, as providing, or even being, a model in the truth-making structure sense of the relevant set of sentences.* After all, the system in question can be said to be a "structure" – it has parts, and those parts have various properties and stand in various relations to one another – and, via a set-theoretical structure which can be constructed from its parts, it provides an interpretation of the relevant sentences on which they come out true.[17] [18]

There is one last way of employing the truth-making structure notion of model which deserves special attention in the present context: a system can be counted as a truth-making structure if it makes true the members of a set of fully interpreted sentences of a language such as ordinary scientific English. Consider, for example, the sentence

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the sense in which a Tarskian model is. (And it is even less to the point to insist that the notion of a mapping be defined set-theoretically – whether the mapping from the parts of a language onto a truth-making structure is a set-theoretical entity has little to do with whether the truth-making structure is.)

* Especially if some intended interpretation of the predicates lies in the background, and using that interpretation with respect to the "structure" in question to determine extensions we get true sentences.... This is related to the next para.

[17] Nagel seems to have an intermediate version of the notion of a truth-making structure in mind when he mentions, as an alternative to the notion he emphasises, that we might think of a certain "*system* of 'things'" as constituting a model for a certain set of postulates (1961, p. 96). The "things" which appear in his example seem to be a set of molecules, the power set of that set, and (a set of?) ratios between various masses. The system in question then does seem to be a set-theoretical entity, but it is not functioning in exactly the same way as $<S_1, S_2, S_3>$ in our initial example. (For Nagel's preferred notion of model, and Braithwaite's, see n. 7.)

[18] Elisabeth Lloyd, for example, uses the term 'structure' in this sense, and opposes our second notion of model, broadened in something like this way, to the notion of a truth-making map. (See the new preface to Lloyd (1994), p. vii.) Note also that her sample sentences (such as "Object A is touching object B"), the ones which truth-making structures interpret and make true, come with interpreted predicates and sortals, and are not written in the vocabulary of any formal language.

(R) Every resistor heats up when a current passes through it.

We can say that a particular circuit is a model of (the singleton of) this sentence if, indeed, every resistor in the circuit heats up when a current passes through it.

This seems to be the notion of model Bas van Fraassen has in mind when he writes, in *Laws and Symmetry*: "A model is called a model of a theory exactly if the theory is entirely true if considered with respect to this model alone. (Figuratively: the theory would be true if this model was the whole world.)" (1989, p. 218). Van Fraassen illustrates this formulation with the example of the Seven Point Space as a model of "Theory *G*," a geometrical theory which he presents by laying out its three axioms. The first axiom, for example ("*A*1"), is that "[f]or any two lines, at most one point lies on both," and the Seven Point Space is then simply a particular arrangement of seven points and seven lines such that for any two of the seven lines, at most one of the seven points lies on both (and similarly for the other two axioms) (1989, pp. 218-19). Here, then, the Seven Point Space can quite naturally classed as a truth-making structure for the set of axioms of Theory *G* – a structure which provides an interpretation for, and makes true, the axioms in the set. And assuming that, for present purposes, we can identify Theory *G* with the set of its axioms, it is thus in the truth-making structure sense of 'model' that the Seven Point Space is model of Theory *G*.[19]

There is an apparent puzzle here, however: If a truth-making structure is a thing which provides an interpretation for a set of sentences (and does so in such a way that sentences come out true on the interpretation), then how can anything be a truth-making structure for a set of *fully interpreted* sentences? The solution to this puzzle is to notice that it trades on an ambiguity in the notion of interpretation. To see this, think carefully about what is going on when we regard a certain electrical circuit as a structure which

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[19] We may prefer to identify Theory *G* with the logical closure of the set of its axioms; but then of course the Seven Point Space can with equal accuracy be described as a truth-making structure for that larger set.

Note also that one might be inclined to think of the axioms of Theory *G*, not as certain fully interpreted sentences, but as the propositions expressed by those sentences. The point discussed in the next paragraph introduces a difficulty for that approach, however.

makes (R) come out true: we are regarding the quantifier 'Every' as implicitly restricted in its scope to the little universe of entities which comprise that circuit. As we move from one circuit to another, asking whether each is a model of (R), we change the domain of discourse, the set of entities over which the term 'Every' quantifies. There are some delicate issues in the semantics of natural language here, but one way of putting the point is as follows. As we move from one circuit to another, the meaning of 'resistor', and of 'current,' and (even, on one approach) of 'Every' is held fixed, and it is in that sense that (R) functioning here as a fully interpreted sentence. (Contrast this with the case of the sentence '(∀x)(Px → Qx)' as we move from one Tarskian structure to another.) But equally, as we move from one circuit to another, the domain of discourse changes, and so (on one approach) the proposition which (R) expresses changes; and in that respect, each individual circuit can be regarded as providing a different interpretation for (R). (Note the *similarity* here to the case of the sentence '(∀x)(Px → Qx)' as we move from one Tarskian structure to another – if providing a Tarskian structure for '(∀x)(Px → Qx)' results in its expressing a proposition at all (see point (ii) in n. 9), then clearly different Tarskian structures yield different propositions.) Some circuits provide an interpretation for (R) on which it comes out true; those circuits are truth-making structures for {(R)}, and so models of it in the present sense.[20]

This is worth spelling out in detail, I think, because it helps us to see something which will prove important in thinking about the best way of understanding the semantic view of scientific theory structure. We can say that $<S_1, S_2, S_3>$ is a truth-making structure for (and so a model of) {(∀x)(Px → Qx)}, and that a certain electrical circuit is a truth-making structure for (and so a model of) {(R)}, and in doing so use the terms 'truth-making structure' and 'model' unambiguously; but there are, nonetheless, important differences between the two cases. $<S_1, S_2, S_3>$ is an object which plays a role in a reasonably elaborate semantical theory of the language in which '(∀x)(Px → Qx)' is

[20] I take it that this analysis of the situation carries over to the case of Theory *G* and the Seven Point Space.

written. It provides interpretations for uninterpreted terms in the sentence in question; and when we say that <S_1, S_2, S_3> is a model of the sentence, we are implicitly invoking the semantical theory, with its notion of interpretation and its definition of truth in an interpretation. The electrical circuit we are imagining, however, does not provide an interpretation for uninterpreted terms in the language of (R), and when we say that the circuit is a model of {(R)}, or a truth-making structure for it, we are *not* invoking any semantical theory (or at any rate, we do not need to be). What we are saying when we say that the circuit is a truth-making structure for {(R)} could also be said simply by saying that the circuit *fits a description* given by (R) – it is a system such that (and here comes (R)…) every resistor heats up when a current passes through it. The same manoeuvre will not work, note, in the Tarskian case: it makes no sense to say "<S_1, S_2, S_3> is an object such that (∀x)(Px → Qx)," because the sentence one will have uttered in saying *that* is itself not fully interpreted.

A first glimpse of the relevance of all this to the understanding of the semantic view comes when we consider Patrick Suppes's version of that view. On Suppes's approach, a theory is presented by the specification of a "set-theoretical predicate"; the theory can then be thought of as the collection of set-theoretical structures picked out by the set-theoretical predicate.[21] Now, it seems to be widely believed that Suppes's set-theoretical predicates stand in the sort of relation to the models they pick out (the models which make up a given theory) that '(∀x)(Px → Qx)' stands in to <S_1, S_2, S_3>. And that belief is part and parcel of a picture according to which Suppes's version of the semantic view, and other versions, too, is employing the Tarskian notion of model – hence, indeed, the term 'semantic view.' (Suppes himself quite clearly seemed to understand the situation this way – see the next section.) I will argue, however, that on reflection it is quite clear instead that a Suppesian set-theoretical predicate stands in essentially the same relation to one of the models of the theory it characterises that (R)

[21] See the discussion of Suppes in the section entitled "Making the most of the semantic view," below.

stands in to an electrical circuit all of whose resistors heat up when subjected to a current – the models simply fit the description given by the set-theoretical predicate.

Mathematical models. The other notion of model we need to spell out in some detail is one which can be extracted from the writings of Patrick Suppes and Bas van Fraassen in their seminal work on the semantic view of theory structure.[22] (For now, I mean to make only the quite minimal claim that one can read certain central passages in the writings of these authors as employing the notion I will characterize. Whether some other way of reading those passages should be preferred is a question which will receive further consideration in the next section.) For example, consider the following excerpt from van Fraassen's 1987 paper, "The Semantic Approach to Scientific Theories":[23]

> [T]he systems [in the domain of inquiry] are physical entities developing in time. They have accordingly a space of possible states, which they take on and change during this development. This introduces the idea of a cluster of models united by a common *state-space*; each has in addition...a 'history function' which assigns to [the modelled system] a history, i.e., a trajectory in that space.

The idea here, then, is that the possible states of a given type of system are represented by points in a mathematical space which in this context we call a *state space*. To take the most obvious example, in classical particle mechanics the state space for a system of n particles will be a $6n$-dimensional vector space, the *phase space*, each point of which corresponds to an assignment of three spatial coordinates (x, y, z) and three components of momentum (p_x, p_y, p_z) to each of the n particles.[24] In orthodox quantum mechanics, on

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[22] The notion also appears in the writings of another originator of the semantic view, Frederick Suppe ((1967), (1974 a), and (1989)), but I will focus on the work of Suppes and van Fraassen here because, as I understand it, Suppe's picture of theory structure is importantly different from those of Suppes and van Fraassen. See Thomson-Jones (forthcoming a) for a related discussion of certain problems with Suppe's framework.

[23] Although I think that the same notion can be found in important early work due to Suppes, there are certain aspects of Suppes's own presentation which would invite confusion at this point, particularly with regard to the essentially straightforward distinction between this new notion of model and the last. For further elaboration of this point, and discussion of Suppes's work, see the next section.

[24] Actually, the retouching I have performed on the van Fraassen quote was designed to remove the suggestion of an alternative approach, on which the state space in this case would be a 6-dimensional phase space regardless of the value of n. The history function then assigns a trajectory through this state space to

the other hand, the state space would be a complex Hilbert space of countably infinite dimensionality. We can define a *trajectory* through the state space to be a function which maps points in some interval of the real line to points in the state space. The points in the domain of such a function are taken to represent times, and so a trajectory represents a particular evolution of the state of the modelled system over time. A model is then simply a state space with a trajectory defined on it.[25]

The notion of model I wish to fix upon is obtained by generalising from the characterization just presented in two simple ways: we will not insist that the systems under study be physical entities, thus making room for an uncontentious application of the notion in the context of economics, for example; and we will not insist that a mathematical object represent the evolution of a system over time in order to count as a model in this third sense.[26] We are left with a notion of model on which a model is simply a mathematical structure used to represent the structure and/or behaviour of a system, or kind of system, from the domain of inquiry corresponding to a given discipline.[27] I will call this sort of model a *mathematical model*.

This label is far from perfect, because the first, second, and fourth categories of model in the taxonomy I am recommending also include objects which could appropriately be described as mathematical. The greatest risk in the present context is that of confusing the notion of a mathematical model, as just defined, with the notion of

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each of the *n* particles ('objects' is van Fraassen's more general term) in the system. This fewer dimensions/more trajectories approach is essentially equivalent to the more dimensions/one trajectory approach I have presented, and the choice between them is likely to rest purely on considerations of mathematical convenience.

[25] Actually, van Fraassen limits this characterization of the notion of model to the case of "non-relativistic" theories (*ibid*.), but as he clearly takes himself to be discussing scientific theories in general, and not just those to be found within physics, this will presumably include the bulk of existing theories (to the extent that a theory in, say, population genetics can be classed as relativistic or otherwise). In this connection, consider the following quotation from Elisabeth Lloyd's *The Structure and Confirmation of Evolutionary Theory* (1994), a book in which she develops the state space version of the semantic view further and then explores various philosophical issues in the foundations of evolutionary biology through its lens: "The models [of a theory] are mathematical models of the evolution of states of a given system, both in isolation and interaction, through time" (p. 19).

[26] In addition to van Fraassen (in the quoted passage) and Lloyd (quoted in the previous note), Suppe imposes a restriction of the sort I mean to lift: "theory structures...specify the admissible behaviors of state transition systems" (1989, p.4).

[27] Van Fraassen, characterizing Suppes's approach in 1972, writes: "From this point of view, the essential job of a scientific theory is to provide us with a family of models, to be used for the representation of empirical phenomena" (van Fraassen (1972), p. 310; quoted in Lloyd (1994), p. 15).

a truth-making structure; keeping those two notions apart will, however, prove vital in the discussion of the proper understanding of the semantic view in the next section. The important thing to remember is that although a mathematical model is, and a truth-making structure can be, a mathematical entity, the associated functions are quite distinct. Truth-making structures interpret and make true sentences; mathematical models represent systems from some given domain of inquiry. A truth-making structure might *also* function as a representation – nothing excludes that – but equally well may not, and it is no part of the notion of a mathematical model as I have defined it that a mathematical model play any role in interpreting or making true any sentences. The conceptual distinction here is clean enough, even if there are objects to which both concepts apply.

Equipped with a clear understanding of these distinct notions of model, we are now in a good position to think about how we should understand the semantic view of theories.

Making the most of the semantic view

Although there are a number of variant versions of the semantic view on the market, the majority of them centrally involve at least one of two claims.[28] First there is:

A scientific theory is a collection of models. (I)

The 'I' here is for 'Identification,' as this claim identifies scientific theories as being certain sorts of object.[29] The second claim is perhaps a less ambitious one, although one

[28] For a detailed history of the semantic view and a survey of the literature on it, see Suppe (1989), pp. 5-20. ***ADD REFS TO PSA 1998 PAPERS, INCL. SUPPES "THIRTY YEARS ON" PAPER, AND ADD TO BIBLIO; AND REFER TO FN. 44 FOR REFS. ON PARTIAL STRUCTURES APPROACH***

[29] See, for example, Giere (1988), pp. 47-8, especially "This makes it possible [for the semantic view] to identify a theory...with [a] set of models." Note, however, that it is harder to find bald statements of such a thesis in the original writings of Suppes or van Fraassen. In his "What Is A Scientific Theory?" (1967), for example, Suppes simply emphasises the value of "extrinsic characterizations" of theories which proceed by picking out a class of models, whilst leaving room for "intrinsic" and more linguistically-oriented formulations as well. Van Fraassen presents the "identification" claim in his "The Semantic Approach to

would not want to say that it is entailed by (I). It is a methodological recommendation directed primarily to those who are engaged in philosophical work on the sciences:

A scientific theory is best thought of as a collection of models. (M)

In both cases, 'collection' is sometimes replaced by 'set,' 'class,' 'family,' or 'population,' and some authors may wish to impose restrictions of scope (to the natural sciences, for example), but whenever possible these differences will be set aside in the present discussion.[30]

A significant twist is due to Ronald Giere, who suggests that we think of theories as having two components, "(1) a population of models, and (2) various hypotheses linking those models with systems in the real world" (1988, p. 85),[31] and van Fraassen embraces Giere's suggestion (1987, p. 109).[32] On the sort of picture which results, the collection of models obviously makes up only one of the two halves of a theory. For simplicity's sake, I will omit this possible emendation to (I) and (M); incorporation would not affect the substance of the discussion.

My initial question, rather unsurprisingly, is now: What notion of model is being invoked when a proponent of the semantic view utters (I), or (M)? As the discussion of this section progresses, we will also consider the distinct and less hermeneutical question: Given the avowed aims of the semantic view, what notion of model should a

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Scientific Theories" (1987, p. 109), but only tentatively, and in the context of describing Suppes's view; his characterizations of the semantic view in *The Scientific Image* (1980, pp. 44 and 64), and in "On the Extension of Beth's Semantics of Physical Theories" (1970), for example, whilst clearly putting the emphasis on models, fall short of making such a claim.

[30] The distinction between (I) and (M), two claims about theories, parallels a distinction between two questions about models to which French and Ladyman draw attention, namely, "What are models?" and "How are they best represented for philosophical purposes?". See ***INSERT REF TO FRENCH AND LADYMAN 1999, ADD TO BIBLIO***

[31] In Giere's hands, the general idea is that the hypotheses in question make claims to the effect that various real systems (or types of real system) are adequately represented by various of the models in certain respects, and to certain degrees; see also Suppes (1967), pp. 62-4.

[32] Although Giere and van Fraassen are working with very different notions of model (see Thomson-Jones, forthcoming a), and as van Fraassen and Giere both recognize, they differ over the typical form of a theoretical hypothesis: for Giere theoretical hypotheses make claims about similarities between models and real systems, whereas for van Fraassen they make claims of isomorphism and embeddability.

proponent of the semantic view be employing in making one or both of the claims in question?

My main thesis can be simply stated: Although the semantic view has often been taken to be the view that theories are, or are best thought of as, collections of entities which are *both* truth-making structures *and* mathematical models, it should in fact be understood as the view that theories are, or are best thought of as, simply collections of mathematical models. The truth-making structure notion of model is best excised from the semantic view altogether (in so far as it is truly present in the first place). The result is a version of the semantic view which is closer to the intentions of the original proponents of the view, despite some of their own declarations to the contrary, and which stands a better chance of achieving the avowed aims of the view.

First, then, to matters of interpretation. I take it to be obvious, and will assume without further argument, that no proponent of the semantic view ever intended to invoke (nor ever invoked) either the notion of a propositional model or the notion of a physical model when putting forth his or her version of (I) or of (M). The notion of a truth-making map can be dismissed almost as quickly: van Fraassen explicitly rejects it on both his own behalf and Suppes's (van Fraassen (1987), p. 109, n. 2); when Suppes invokes a logical notion of model he usually mentions Tarski, whose notion of model, as we have seen, was that of a (set-theoretical) truth-making structure, rather than that of a truth-making map (see especially Suppes (1960)); and van Fraassen (*loc. cit.*), Suppes (1967, p. 57), Suppe (1989, p. 4), and Giere (1988, pp. 47-8) all insist that models in the relevant sense are nonlinguistic (or "extralinguistic") entities,[33] whereas a truth-making map is a partially linguistic thing, for it is a function which takes the non-logical vocabulary of a particular language as its domain.

[33] Confusingly, Giere's emphatic claim to this effect follows immediately upon an Enderton-style definition of 'model' which makes models truth-making maps (*op. cit.*, p. 47). I think this must be read as an error on Giere's part.

The models on which Suppes and van Fraassen place the most emphasis are mathematical structures of various kinds. One might thus be tempted to ask, in light of the taxonomy laid out at the beginning of the discussion, and the options eliminated thus far, whether these mathematical structures, the models of which the semantic view theorist speaks, are to be thought of as truth-making structures of some specific variety, or as mathematical models – i.e., as representations of systems from the domain of inquiry. That question presents a false dichotomy, however. In particular, a third option is that when a proponent of the semantic view puts forth (I), or (M), she means to use the term 'models' to invoke what we might call a "double aspect" conception of model – that is, it may be that she means to refer to entities which function *both* as truth-making structures and as mathematical models in the representational sense. And as I have already mentioned, I take it that this is just how many people understand the semantic view.

It is quite clear, I think, that the models of the semantic view theorist are intended to function as representations.[34] What is less clear is the extent to which the models referred to in (I) or (M) are also intended to be functioning also as truth-making structures. On the one hand, both Suppes and van Fraassen make explicit statements to the effect that the notion they wish to employ is, or is very closely related to, a logical notion,[35] and van Fraassen begins his presentations of the semantic view in both *The Scientific Image* (1980) and *Laws and Symmetry* (1989) by introducing a notion of model on which models are clearly functioning as truth-making structures in one way or another.[36] On the other hand, when it comes to showing the naturalness and plausibility with which theories in the empirical sciences can be viewed as collections of models, it is quite unclear that the models in question are, as constituents of those theories,

[34] See the following discussion of Suppes, and the discussion of van Fraassen on mathematical ("state-space") models, above.
[35] Suppes (1960), p. 289, and (1967), p. 57; van Fraassen (1980), p. 44.
[36] Van Fraassen (1980), pp. 41-44, and (1989), pp. 217-220. In both cases, van Fraassen draws an example from logical and mathematical work in the foundations of geometry; indeed, the discussion in (1989) is the source of the 'Theory *G*' example discussed earlier.

functioning as truth-making structures in any interesting way. There is ample room for confusion on this score, however, and so it will be worthwhile to take a moment and consider an example in detail.

A good example of the way in which Suppes makes the case that theories can be thought of as collections of models is provided by his 1957 “axiomatization” of Newtonian particle mechanics (1957, p. 294):[37]

> DEFINITION 1. A system ß = <P, T, s, m, f, g> is a [model] of particle mechanics if and only if the following seven axioms are satisfied:
>
> KINEMATICAL AXIOMS
>
> AXIOM P1. The set P is finite and non-empty.
> AXIOM P2. The set T is an interval of real numbers.
> AXIOM P3. For p in P, s_p is twice differentiable on T.
>
> DYNAMICAL AXIOMS
>
> AXIOM P4. For p in P, m(p) is a positive real number.
> AXIOM P5. For p and q in P and t in T,
>
> $$f(p, q, t) = -f(q, p, t).$$
>
> AXIOM P6. For p and q in P and t in T,
>
> $$s(p, t) \times f(p, q, t) = -s(q, t) \times f(q, p, t).$$
>
> AXIOM P7. For p in P and t in T,
>
> $$m(p)D^2 s_p(t) = {}_{q \in P} f(p, q, t) + g(p, t).$$

In presenting a theory this way, we are introducing what Suppes calls a “set-theoretical predicate” (1957, ch. 12, esp. §12.2).[38] In this case the set-theoretical predicate is ‘is a

[37] The word ‘model’ replaces the word ‘system.’ Suppes himself does not use the word ‘model’ much in the (1957) discussion; nonetheless, his remarks at (1957), p. 299, and his references back to the 1957 discussion on p. 291 of his (1960) make it clear that he wishes to apply the term ‘model’ to the “systems of particle mechanics” defined in the quoted passage.

For more examples of Suppesian “axiomatizations” of scientific theories, see, for example, Suppes (1974). My use of scare quotes here will be explained shortly.

[38] Suppes is taking it that the notions of real number, vector, function, derivative of a function, and so on have all, at least in principle, been given a set-theoretical analysis in the context of an axiomatic set theory. See (1957, pp. 249-50).

[model] of particle mechanics.'[39] We then pick out a class of set-theoretical entities simply by instructing our audience to consider the class of objects to which the predicate applies, or, to put it another way, which satisfy the predicate. And the fundamental idea behind the semantic approach to scientific theories, in Suppes's version of that approach, is a version of the idea expressed by (M): it is that philosophers of science will most readily gain insight into the structure of scientific theories if they think of them as the sorts of things which can be presented in just this way – that is, by way of the definition of a set-theoretical predicate.

Importantly, and as is often stressed, this method of picking out a class of set-theoretical entities allows us to avoid any reference to a set of sentences in any particular language, or any invocation, implicit or otherwise, of a model-theoretic semantics. Thus, in characterizing Suppes's proposal, van Fraassen writes approvingly: "*to present a theory, we define the class of its models directly*, without paying any attention to questions of axiomatizability, in any special language" (1987, p. 109, emphasis in original). Indeed, not only is there no explicit mention of any formal language, or any set of sentences in such a language, but it is quite unclear what set of sentences in what formal language would do the trick, via a corresponding truth-making relation, of capturing the class of structures the definition picks outs.[40] And this is in no way a defect of the definition as a means of presenting Newtonian particle mechanics, *if* the point is only to carve out a class of mathematical structures (the models of the theory, in our present sense) which, according to the theory, are adequate in one way or another to the job of representing

[39] To avoid confusion, it is important to bear in mind that, absent a thoroughgoing nominalism of exactly the right sort, no system of particles enduring over time could count amongst the "system[s] of particle mechanics," as that phrase is defined in the original, unmodified version of Suppes's Definition 1, for the latter are ordered tuples whose elements are scalar-valued functions, vector-valued functions, sets of real numbers, and the like. It is thus an unfortunate choice of words to call the objects to which the set-theoretical predicate applies the "systems of particle mechanics"; in addition to being true to Suppes's intentions, "models of the theory of particle mechanics" then seems a less troubled choice.

[40] This claim is in no way compromised by the fact that Axioms P5-P7 correspond in fairly straightforward ways to Newton's laws of motion. (See Suppes's discussion at (1957), pp. 296-8.) For one thing, it is clear that the class of set-theoretical structures picked out by Definition 1 is not the class of truth-making structures of any set of sentences in a denumerable first-order language (i.e., it is not an elementary class); to see this, just consider Axiom P2 in the light of light of the Löwenheim-Skolem theorem. (Van Fraassen makes this point in a different setting in his (1985), pp. 301-2; see also his (1987), p. 120.)

actual or possible collections of particles as they move under the influence of various forces.

It is important not to be misled by Suppes's own use of the word 'axiomatize.' Suppes is not offering an axiomatization of that theory in the sense in which a proponent of the syntactic view would seek one (although that is presumably the sense van Fraassen has in mind in his characterization of Suppes's idea). One of the "axioms" Suppes presents, for example, is that 'T', the second element of any tuple which is to satisfy the set-theoretical predicate in question (and so count as a model of the theory) must be an interval of real numbers (Axiom P2). Clearly, then, the "axioms" Suppes presents do not form a set of sentences whose deductive closure can be regarded as capturing the content, or even the logical structure, of Newtonian particle mechanics, for then they would have to be about (or translations into a formal language of sentences about) forces, masses, accelerations, and the like; axioms in the sense Suppes employs are simply components of a characterization of a class of mathematical structures.

Correspondingly, although we might regard a given model in Suppes's sense (i.e., a given mathematical structure satisfying the predicate 'is a system of particle mechanics') as a truth-making structure for sentences such as 'The set *P* [or: The first set in the tuple] is finite and non-empty' and 'The set *T* [or: The second set in the tuple] is an interval of real numbers,' in the same sort of way that an electrical circuit can be regarded as a truth-making structure for the sentence 'Every resistor heats up when a current passes through it,' this is in essence, as we have seen, simply to say that it fits the description we have given in our attempt to pick out a certain kind of mathematical structure. And that does not mean that any given model of the theory in Suppes's sense is a truth-making structure for some set of sentences which we might regard as a linguistic formulation of the classical theory of particle mechanics. Given all this, it is hard to resist the conclusion that the label 'semantic view' (and the label 'model-theoretic view,' for that matter) is as much a misnomer as the name 'received view' has become.

Van Fraassen makes this point about the different senses of 'axiomatize' quite forcefully ((1980), p. 65; see also (1970), p. 337), and in the same setting describes his own picture of theory structure as closely allied to Suppes's:

> To present classical mechanics, for instance, [Suppes] would give the definition: 'A system of classical mechanics is a mathematical structure of the following sort...' where the dots are replaced by a set-theoretic predicate. Although I do not wish to favour any mathematical presentation as the canonical one, I am clearly following here his general conception of how, say, the theory of classical mechanics is to be identified. (1980, p. 66)

The conclusion I would wish to draw is that, insofar as claims (I) or (M) are supported by the ease with which we can give presentations of empirical theories on which they look like collections of models, the appropriate notion of model is simply that of the mathematical model. The mathematical structures which such presentations pick out are clearly meant to be used for the purposes of representation; but there is no special reason for supposing that they play a role as truth-making structures for any linguistic formulation of the relevant theory.

Reading (I) and (M), the defining theses of the semantic view, as involving only the notion of the mathematical model, so that all talk of truth-making falls away, fits nicely with Suppes's slogan that "philosophy of science should use mathematics, and not meta-mathematics" (van Fraassen, 1980, p. 65), for model theory is surely meta-mathematics. Such a reading also seems to make the most sense of van Fraassen's claims that the semantic view is "a view of theories which makes language largely irrelevant to the subject," and that, on the semantic view, "models are mathematical structures, called models of a given theory only by virtue of belonging to the class defined to be the models of that theory" (1987, p. 108 and p. 109, n. 2).

Most importantly, however, it would seem that employing only the notion of a mathematical model in advancing (I) or (M) would be the best way to vouchsafe the attainment of certain avowed goals of the semantic view. An account of theory structure, I take it, derives its primary philosophical value from the accounts it facilitates of confirmation, explanation, theory-testing, and the like, and from the light it thus sheds on debates over realism and rationality. Proponents of the semantic view have repeatedly claimed that their view of theory structure has at least two advantages when it comes to this sort of philosophical work: that by construing theories non-linguistically it allows us to sidestep a slew of familiar problems about the meaning, reference, and classification of various terms in the language scientists employ, and that it leads to a way of thinking about theories which maintains greater closeness to scientific practice than the older, syntactic view. Given this, it seems that a proponent of the semantic view ought to prefer the notion of a mathematical model to either the notion of a truth-making structure, or a hybrid notion on which models play both truth-making and representational roles, for if the models to which (I) and (M) refer are to be thought of as functioning crucially as truth-makers for sentences making up some linguistic formulation of the theory, there is surely a significant danger that we will begin to focus once more on the sentences in questions and the language in which they are written[41]; and as we do so, we are likely to drift further and further away from scientific practice.

Overall, then, it seems to me that the semantic view has little to lose, and something to gain, by employing in its central theses a concept of model shorn of the trappings of model-theoretic semantics, and the role of truth-making.[42] We should set

[41] Accordingly, it will also be difficult to get philosophers of science to stop thinking of laws as fundamental to theories, something van Fraassen, for one, wants very much to do. See his (1989).

[42] A notable exception to this general claim is perhaps the sort of work in which van Fraassen was engaged in his (1970). (See also van Fraassen (1972).) There the state space for a system, which clearly plays a representational role, also plays a role in the proposed formal semantics for the language of the given physical theory. The project, however, is first and foremost a logical one, and is described as such; and interestingly, van Fraassen makes only the importantly qualified claim that the picture of theory structure he presents is "more faithful to current practice *in foundational research* in the sciences than the familiar picture of a partly interpreted axiomatic theory" (1970, abstract, emphasis added; see also pp. 337-8). (The

aside the "double aspect" notion of model, and drop altogether the idea that the models which compose a theory play some non-trivial truth-making role, or that regarding them as doing so is an important part of understanding scientific theories. The notion of a mathematical model – that is, of a mathematical structure which functions as a representation of systems from the domain of inquiry – is all we need to obtain the most attractive and useful version of the semantic view.

The implications for the semantic view

Understanding the semantic view in the way I am recommending leaves us with a considerably more flexible approach to understanding theory structure than we must take, at least in practice, if the models of the semantic view are to function as truth-making structures in some non-trivial sense. There are many sorts of mathematical structure, and many ways in which those many kinds can, and do, serve representational ends. We will thus have, as philosophers of science, a rich palette at our disposal when we turn to understanding the details of theory structure, the relationship of theories to the phenomena, inter-theory relations, and related topics such as confirmation, explanation, and the rest, if we allow ourselves to draw on the great variety of mathematical structures there are in constructing our accounts. But if we insist on regarding scientific theories as collections of objects which, in addition to functioning as representations, also play a substantial role in some semantical theory – objects which are, non-trivially, truth-making structures – we will, in practice, end up limiting ourselves to thinking in terms of the sorts of mathematical structures which have been employed in the semantical theories we have managed to construct. Such an approach, I would claim, carries with it extraneous baggage, makes the task more difficult than it need be, and lessens the chances of ultimate success.

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discussion makes it clear that work in quantum logic is the kind of research he has in mind.) One might simply see this as evidence that different philosophical projects call for different notions of model.

A specific corollary of this general point is that there is no obvious reason to insist, *ab initio*, on employing set-theoretical tools in pursuing the philosophical "research programs" associated with the semantic view. There is a firmly entrenched model-theoretic approach to the semantics of a range of formal languages which is, of course, heavily set-theoretical, and models in Tarski's sense are set-theoretical structures par excellence; but on the approach to the semantic view I am recommending, those facts become irrelevant to the question of our choice of mathematical framework in analysing theory structure. We should be careful, too, not to overstate the extent to which de-emphasising set theory involves a divergence from the approach taken by the original architects of the semantic view. Consider, for example, van Fraassen's state-space models: the fact that a vector space, and a trajectory through one, can both be treated as set-theoretical objects plays no obvious role in accounting for the representational capacities of the model, or its role as one of the structures which makes up the theory. And even though on Suppes's "axiomatization" of classical particle mechanics the models of the theory are explicitly presented to us as set-theoretical entities, that fact is not especially important if we are thinking of those models simply as tools for the representation of physical systems – we need pay no particular attention, for example, to the fact that the notions of a tuple, a function, a vector, and the reals can be given set-theoretical definitions.[43]

Amongst the more recent versions of the semantic view, the "partial structures" approach developed and extended by Newton da Costa, Steven French, Otávio Bueno, James Ladyman, and others seems most emphatic about a reliance on the tools of set theory, and on the use of just the sorts of set-theoretical structures which have been employed in semantical theory.[44] (Of course, one of the innovations of this approach is

[43] It is interesting, to note, too, that the tuples Suppes focussed on do not include a universe of discourse containing the ur-elements out of which the other sets in the tuple must be constructed, unlike Tarski's truth-making models. It would be a trivial formal manoeuvre to add one, of course, but it is perhaps telling that Suppes did not, despite his insistence that he was using Tarski's notion of model (Suppes (1960), p. 289).
[44] See, for example, ***INSERT REFS., AND IN BIBLIO***

the replacement of total relations with partial relations, and, as a consequence, of Tarskian total structures with partial structures, so the fundamental entity of the partial structures approach is not *exactly* the same kind of entity as the one we are most familiar with from the standard model-theoretic semantics of first-order languages; nonetheless, the partial structure is clearly a very close cousin of the Tarskian model.) Indeed, given my reading of Suppes and of van Fraassen, there is in fact a tighter connection between the semantic view and work in formal semantics on the partial structures approach than was present in the original formulations of the view. It does not follow from this, however, that there is anything wrong-headed about the partial structures approach, understood as one version of the semantic view. For all I have said here, we might even conclude, at the end of all inquiry, that the partial structures approach is the best way of developing the semantic view. The proof may be in the pudding – a pudding (if the friends of partial structures will forgive me the metaphor) which already includes accounts of empirical adequacy, of the relation between theories and data models, of the nature of iconic models, and more. The only point I would wish to make here is that there is, again, no *a priori* reason, so to speak, to think that we ought to focus on partial structures or anything like them in taking the semantic view of theory structure, or in developing it. Whether we have already been given a *post facto* justification for doing so, in the work that has been done on the partial structures approach, is a matter for separate discussion.

Scientific structuralism and structural realism

The semantic view looks best, then, I claim, if it is taken to be the view that theories are collections of mathematical models, rather than of "double aspect" models which play both representational and truth-making roles. If embracing this understanding of the semantic view strengthens it, as I have argued, then it would also seem to lend some support to the view that all scientific representation is structural – after all, the semantic view provides an account of one central sort of scientific representation, the theory, according to which it is a collection of structures, and so in making the semantic view seem as plausible and defensible as possible by insisting on a more flexible notion of structure, we can only be improving the prospects for a general structuralism (whilst still, I think, invoking a sufficiently strong notion of structure to make it an interesting view).

It is worth distinguishing two structuralist theses at this point. One is the view that all scientific representation is *by means of* structures – call this "vehicle structuralism." Another is the view that scientific representation is only ever *of* structure – call this "content structuralism."[45] Of these two theses, the defence of the semantic view lends the most immediate support to vehicle structuralism, I take it. Now content structuralism is surely neither presupposed nor entailed by vehicle structuralism: structures might be used to represent something other than the structure of the world, and (assuming that the notion of a structure is thick enough to allow that there are things which are not structures) we might use something other than structures to provide structural information. Nonetheless, it seems natural to assume that an argument for vehicle structuralism lends *some* support, indirectly, to content structuralism. So if I am right about the semantic view, we seem, *prima facie*, to have some support for scientific structuralism of both varieties.

There are two last points that I would like to mention, however. The first is that if content structuralism is right, then the defenders of structural realism would seem to face a problem. There is, of course, no direct contradiction between the claim that scientific representation only ever tells us about the structure of the world, and the central structural realist thesis that we have good reason to believe what our best

[45] I will assume for the sake of argument here that there is some serviceable distinction between the structure of some part of the world and what we might call, for want of a better label, the "specifics." (It would be confusing at this point to oppose the structure to the "content" of some part of the world, as we are already speaking of the content of a representation, and allowing that that content might include claims about both the structure of the world and about…well, the specifics.)

theories tell us about the structure of even the unobservable world (and no more). But the main *argument* for structural realism is that it is the only position which both does justice to the intuitions underlying the miracle argument, and escapes the jaws of the pessimistic induction.[46] The alleged evasion of the pessimistic induction, however, relies on the idea that theories tell us both about the structure of the world, and about the specifics: the radical discontinuities we see between successive theories in the history of even the mature sciences are, we are told, discontinuities only with regard to the specifics; at the level of postulated structure, there is enough continuity to make realism plausible. If content structuralism is correct, however, so that everything scientific theories say is about the structure of the world, then the radical differences of content that obtain between, say, Fresnel's aether theory of light and Maxwell's electromagnetic theory (to use Worrall's example) will have to be radical differences of postulated structure. At the very least, then, the structural realist would have to provide us with a distinction between two kinds of postulated structure – the kind we should believe in, and the kind we should not.

There is thus some reason to think that the structural realist will do best to reject content structuralism. That may be harder to do the more defensible the semantic view seems; but for one thing, there is the option of accepting vehicle structuralism and rejecting content structuralism. Though the resulting picture of representation in scientific theories does not by itself seem in any obvious way to lend special support to structural realism as opposed to, say, constructive empiricism,[47] it would avoid the problem I have just described.

Finally, there is a twist in my tale: I should make it plain that I have, in this paper, been playing for the other side. Although I have been describing the way in

[46] ***REF TO WORRALL, AND IN BIBLIO***

[47] Nor, it seems to me, does the picture of representation in scientific theories which results from the acceptance of both vehicle and content structuralism make either structural realism or constructive empiricism seem more plausible than the other.

which, it seems to me, the semantic view of scientific theories can be made strongest, my overall view is that the best philosophical account of such central scientific representations as the Bohr model of the hydrogen atom, and the nuclear model of the cell, involves thinking of such representations as *propositional* models. And, relatedly, I think it is at least worth exploring the idea that scientific theories might fruitfully be regarded as collections of propositional models.[48] The resulting picture of scientific representation is one on which not all representations would be structures, or at least not in the sense proponents of scientific structuralism typically have in mind; there is thus, on such a picture, one less reason for thinking that all scientific representation is of structure. Accordingly, such a picture of scientific representation is more hospitable than an austerely structuralist conception to the idea that we can coherently take science both to represent the world as having a certain structure, *and* to specify the sorts of things, properties, and relations which stand in the postulated structural relations; thus it is also, perhaps, more hospitable to structural realism. But that is a story for another day.[%]

[48] See Thomson-Jones (forthcoming b).

[%] My views on this score seem at least complementary to Anjan's in his (2001), and so I'll ultimately need to think through Steven and Juha's response to Anjan in their paper for this symposium. But that's work in progress.

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