Jack S. Calcut Teaching Statement

I teach with passion and energy, trying not to forget that:

> ...once your mind switches from the state of darkness to the light, all memory of the dark state is erased and it becomes impossible to conceive the existence of another mind for which the idea appears nonobvious. -Mikhael Gromov 1992

Friendly concepts such as compactness and transversality were once foreign to us all and are strangers to our students. I strive to remember this fact, for it is easy to forget the difficulties we ourselves encountered and overcame.

I work hard at planning lectures that motivate the material, promote conceptual understanding, and contain rigorous mathematical definitions and logical reasoning. Below are examples of techniques I employ to attain these goals.

October 12, 2009

Currently, I am teaching two small sections of linear algebra at Michigan State University. This course was recently designated the first proof course at MSU, replacing abstract algebra in this role.

My goal on October 12 was to prove that the span of a given list of vectors in V may be characterized as the smallest subspace of V containing the list. An easily overlooked difficulty here is: *what does smallest mean?*

The lecture began with me briefly recalling the usual total order on the real line, something students are familiar with at least conceptually. Next, we recalled the precise definition of an interval in the real line. I then asked students: *what is the smallest open interval containing 0?*

Students generally agreed that no such interval exists. Next, I asked: *how should we define what it means for one interval to be smaller than another?* Students gave several interesting answers including: compare lengths, compare cardinalities, compare smallest elements, and even inclusion (eureka!). We briefly discussed some pros and cons of each answer and agreed to focus on inclusion. We defined $I \leq J$ to mean $I \subseteq J$ and then discussed the apparent shortcoming of this definition, namely that some intervals are incomparable. I mentioned that we have a partial order, although I resisted the urge to delve into posets formally.

I then gave a general, working definition for the smallest (resp. largest) object satisfying some conditions C, namely: (i) it is an object X satisfying C, and (ii) if Y is any object satisfying C, then $X \leq Y$ (resp. $X \geq Y$). A few real-world examples followed, including the highest scoring freshman in basketball, along with a series of mathematical concepts that each adhered to this general definition. These included the gcd, lcm, supremum (lub), and infimum (glb). We discussed how the precise definition of each of these objects follows exactly the format of our working definition of smallest/largest. At MSU, many mathematics majors take real analysis after linear algebra, and so I also hoped this first exposure to the supremum, as a special case of a general way of thinking, might be helpful to students in the future. I closed our general discussion by pointing out that the concept of smallest/largest that we developed pervades many branches of mathematics including algebra, discrete mathematics, topology, and algebraic geometry.

Finally, we moved to our main problem of proving that the span of a list of vectors in V is the smallest subspace of V containing the list. Students broke into small groups of two to four and worked on a proof. After a few minutes, individual students came to the board voluntarily and presented steps in their arguments. As a class, we arrived at a rigorous proof. I believe students left with more than a proof, namely with a conceptual understanding of smallest/largest in situations where not all objects are comparable and with an idea of how this may fit into their future mathematics courses.

Fall 2006

During the Fall semester of 2006 at the University of Texas, I taught large lecture differential equations, small lecture discrete mathematics, and an independent study on writing in mathematics.

I began my course on differential equations by asking students: *what is an equation and what does equals mean?* These naïve questions lead to important concepts such as types of equations (e.g., definitions, identities, functional, differential, and so forth), implied meaning in equations, and the notion of equivalent instead of equals. Students grasp the example \$1=100¢, where equality is not always intended. Equality of functions versus equality of their defining expressions is another subtle issue that I raise in appropriate courses. Posing the above questions served two purposes: to engage students immediately in mathematical discussions and to point out key concepts for the course.

These questions were followed by a rapid review of calculus with an emphasis on the mean value theorem. Indeed, students already know existence and uniqueness of solutions to many differential equations by the MVT. I believe this point is worth emphasizing for three reasons: (i) to connect the current course with their past work, (ii) to demonstrate the importance of the MVT, and (iii) later in the course, existence and uniqueness is nontrivial and sometimes mysterious for students.

I gave a detailed proof that rates add (e.g., in flow and interest problems) just like vectors. This fact is glossed over in many texts, but is extremely important. First, the class solved two simpler problems: (i) continuous investment with no interest and (ii) continuously compounded interest on a lump initial investment. Next, for continuous investment with continuously compounded interest, the rate of change in the total account value is the sum of the rates in (i) and (ii). To show this, we bounded the difference quotient and obtained the derivative using the squeeze theorem and a limit. I included a discussion of "Big-O" notation for simplifying derivative computations where higher order terms do not matter. This concept is very useful in mathematics and physics. In the end, we saw that indeed the rates do add. Along the way, students learned how to produce a governing ODE using a limiting process.

In the midst of the computation of the derivative above, a light bulb went on for one student who interrupted and asked: *what about the interest on the interest, and the interest on the interest on the interest, and...?* (eureka!) Even though I had spent time motivating the problem, it was not until some time later during the proof that the student began to understand the problem. We took a few minutes to discuss the student's revelation, after which I reiterated the motivation of the problem and completed the computation.

After solving the usual second order, linear, homogeneous, constant coefficient equation, I spent a whole week on a tangible problem, the ideal simple pendulum, covering 14 pages of my own handouts[1]. My notes approach the pendulum in a natural, exploratory fashion and reveal the following: a basic physical situation whose governing ODE already cannot be solved by elementary functions[2], linearization, stability/instability of equilibrium solutions, numerical solutions, approximations, existence/uniqueness and relevance to real life (determinism), real solutions via complex numbers, hyperbolic trigonometric functions arising naturally, and some math history via Galileo. Students love learning that Galileo was wrong about the period of the pendulum, contrary to what is usually learned in physics 101. I am quick to remind them that we have the powerful tools of calculus and computers at our disposal.

[1] See http://www.math.msu.edu/~jack/ for these handouts.

[2] I also like to ask students to compute the length of the graph of one period of the sine function. Incidentally, it is interesting to note that this length is not much greater than the straight walk (why?). Often the graph of sine is drawn poorly scale-wise.

Concluding Remarks

I enjoy and, in fact, prefer teaching all kinds of mathematics courses. I feel that teaching a variety of courses keeps my presentations fresh. I have experience teaching calculus (several forms), differential equations, analysis, probability, applied linear algebra, discrete mathematics, abstract algebra, modern geometry, mathematics for elementary education majors, and independent studies in topology and mathematical writing. Whenever possible, I relate my lessons to other branches of science including physics, astronomy, computer science, and chemistry. I have also had the pleasure of leading several undergraduates in research, including two who obtained publishable results.

I have performed various services for the mathematics departments at the University of Maryland, UT, and MSU. For example, I participated in Building Learning with Technology[3] workshops at UMD in a team led by Karen McLaren where we incorporated Geometer's Sketchpad into the geometry portion of mathematics for elementary education majors. I also presented a popular lecture at UT to middle school students, high school students, and their parents on relativity and 4-dimensional geometry. Most recently, I wrote the uniform calculus final examination and led the mass grading session at MSU during the Spring semester 2009 at the request of Milan Miklavcic, the associate undergraduate chair.

I strongly advocate physical demonstrations in the mathematics classroom. Don't we all think back on actually playing with a spinning top as a child when we prove it is stable? I plan to assemble small, interdisciplinary teams of undergraduates to build devices for classroom use. For example, I have designed a device where two pendulums with rigid rods swing on ball bearings on a single axel, one in front of the other, including a digital interface with a computer. Such a pendulum will facilitate many discussions and provide visual demonstrations of the falsity of Galileo's observation on period.

My ongoing goals are to maintain my energy in the classroom and to reach individuals. A past student of mine, Raudy Perez, struggled on the first analysis midterm, but then began to work feverishly. He was in my office multiple times each week and gained a thorough understanding of the material. He earned his A and went on to pursue a career teaching secondary mathematics. I hope his struggle with and subsequent victory over compactness permeate his teaching. Raudy continues to keep in touch with me five years after our course together.

I am always reminded of Gromov's dictum that once in the light, it is difficult to remember what the darkness was like. I take this dictum with me into the classroom, along with energy and realism, since there are uneventful days teaching, such as days full of definitions. Indeed, not every day can be above average by the mean value theorem. However, there is nothing quite like a day when the motivating examples, the definitions, and the lemmas crystallize for students into a main result.

[3] See http://www.education.umd.edu/blt/.