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**Dugundji, J. [Dugundji, James]**

**Modified Vietoris theorems for homotopy.**

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S. Smale's Vietoris theorem for homotopy [Proc. Amer. Math. Soc. **8** (1957), 604–610; MR0087106 (19,302f)] imposes local connectivity conditions on the fibers of the given map $p\colon X \to Y$; the present paper offers versions that depend on the manner in which the fibers of $p$ are embedded in $X$, rather than on their actual structure. These versions result from a careful study of the homotopy condition on the embedding of the fibers which was considered by T. M. Price [Notices Amer. Math. Soc. **14** (1967), 274, Abstract 67T-197]: A subset $A$ of a $T_2$ space $X$ is called $\mathrm{PC}_X{}^n$ if for each neighborhood $U$ of $A$ in $X$ there is a neighborhood $V$ of $A$, $V \subset U$, such that each map of an $r$-sphere into $V$ has an extension mapping the $(r+1)$-cell into $U$, $0 \leq r \leq n$.

Applications include a proof of the generalization of Smale's theorem announced by G. Kozlowski [ibid. **15** (1968), 560, Abstract 68T-406] plus the theorems quoted below, on homotopy excision and on Serre fibrations:

Let $X$ be paracompact and $A \subset X$ a closed $\mathrm{PC}_X{}^n$ subset. Let $p\colon X \to X/A$ be the projection. If $X/A$ is dominated by a polytope, then $p_*\colon \pi_i(X) \to \pi_i(X/A)$ is an isomorphism for $0 \leq i \leq n$ and is epic for $i = n+1$.

Let $E$ be compact, $B$ a polytope and $(E, p, B)$ a Serre fibration. Then every fiber is $\mathrm{PC}_X{}^n$ if and only if every fiber is $n$-connected.

Reviewed by *George McCarty*