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CHAPTER 7

Problems on singularities and dynamical systems

V. ARNOLD

This paper more or less reproduces the 1991 list of problems of the Singularity Seminar at Moscow State University. Some of the previous lists (compiled every year) have been published in [1–4]. I.G. Petrovsky once said that he had succeeded in doing something in mathematics more because he had not been aware of many things than because he knew something. However, he added, it was always very important to know, that something is not known.

FAKE $\mathbb{R}^4$ AND DYNAMICAL SYSTEMS

Is it possible to write down explicitly a vector field in $\mathbb{R}^5$ (say, with polynomial or trigonometric polynomial components), whose orbit space is a fake $\mathbb{R}^4$?

Let us consider the product $X \times \mathbb{R}$, where X is a manifold, homeomorphic to $\mathbb{R}^4$ but not diffeomorphic to it. The product is diffeomorphic to $\mathbb{R}^5$. Let us consider the vector field $v = (0, 1)$ on the product space. The orbit space of v is X. What are the restrictions on the components of the vector field v in the usual coordinates of $\mathbb{R}^5$, imposed by the fact that X is fake? Can these components be polynomials or trigonometric polynomials?

LIMIT CYCLES AND ELLIPTIC CURVES

The elliptic curve, associated with a real limit cycle of a holomorphic plane vector field, is the local orbit space in the complex neighbourhood of the cycle.